第１０回QUEST研究会
応用力学研究所会議室
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QUESTにおけるプラズマ形状の再構成高精度化

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QUESTにおけるプラズマ形状の再構成高精度化

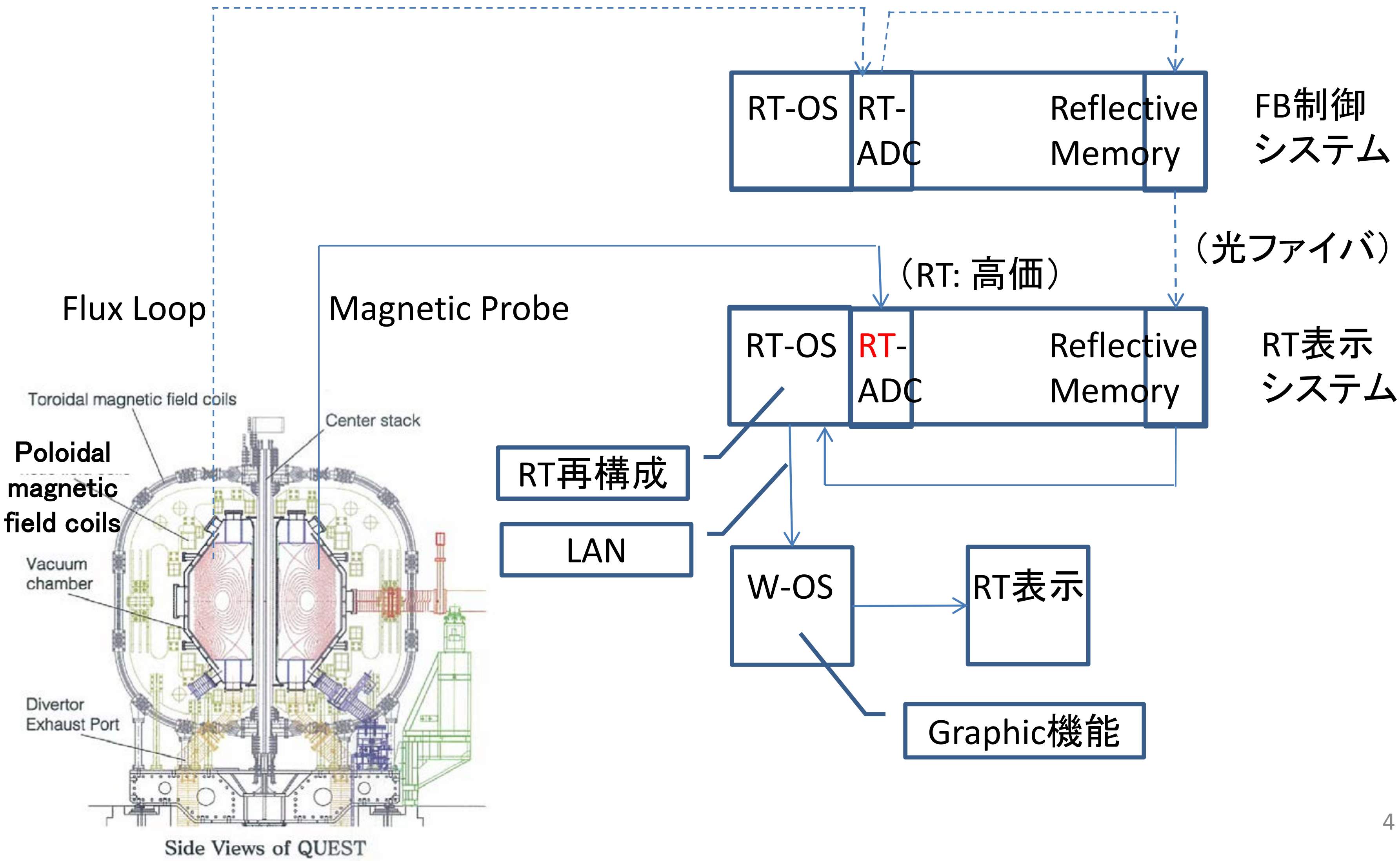
1. 目的
2. 再構成と実時間表示システム
3. 再構成高精度化と実時間表示
4. 研究結果
5. まとめと今後の予定
6. 研究成果

１．目的

　ＲＦ非誘導電流駆動のダイバータ運転シナリオとして、「ＣＳ有りのプラズマ電流立上げとＲＦ電流維持」を実現するには、プラズマ断面形状の（実時間）再構成および実時間フィードバック制御が必要不可欠であり、高温壁やダイバータ配位を用いたプラズマ壁相互作用（ＰＷＩ）の研究にはこれらの高精度化が重要である。

　本研究の目的はダイバータ配位プラズマ断面形状の再構成高精度化を図るとともにその実時間フィードバック制御システムを発展させ、今後のQUESTのミッション達成を可能にすることである。

2. 再構成と実時間表示システム



３．再構成高精度化と実時間表示

- フラックスループと磁気プローブ
- 絶縁アンプ
- 実時間AD変換器
- 積分ドリフト対策（RTを配慮）
- CCS（コーシー条件面）法
- 渦電流の考慮
- SVD, GCV

- リフレクティブメモリ（RT: 高価）
- Windows OS
- Graphic Software

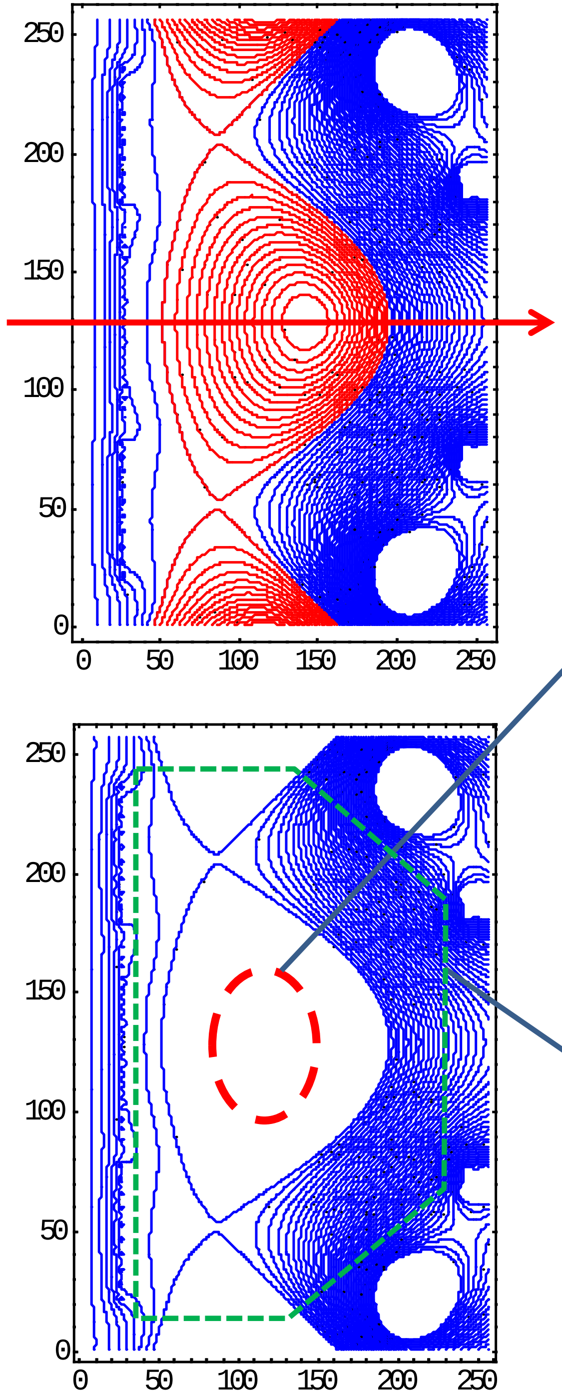
４．研究結果

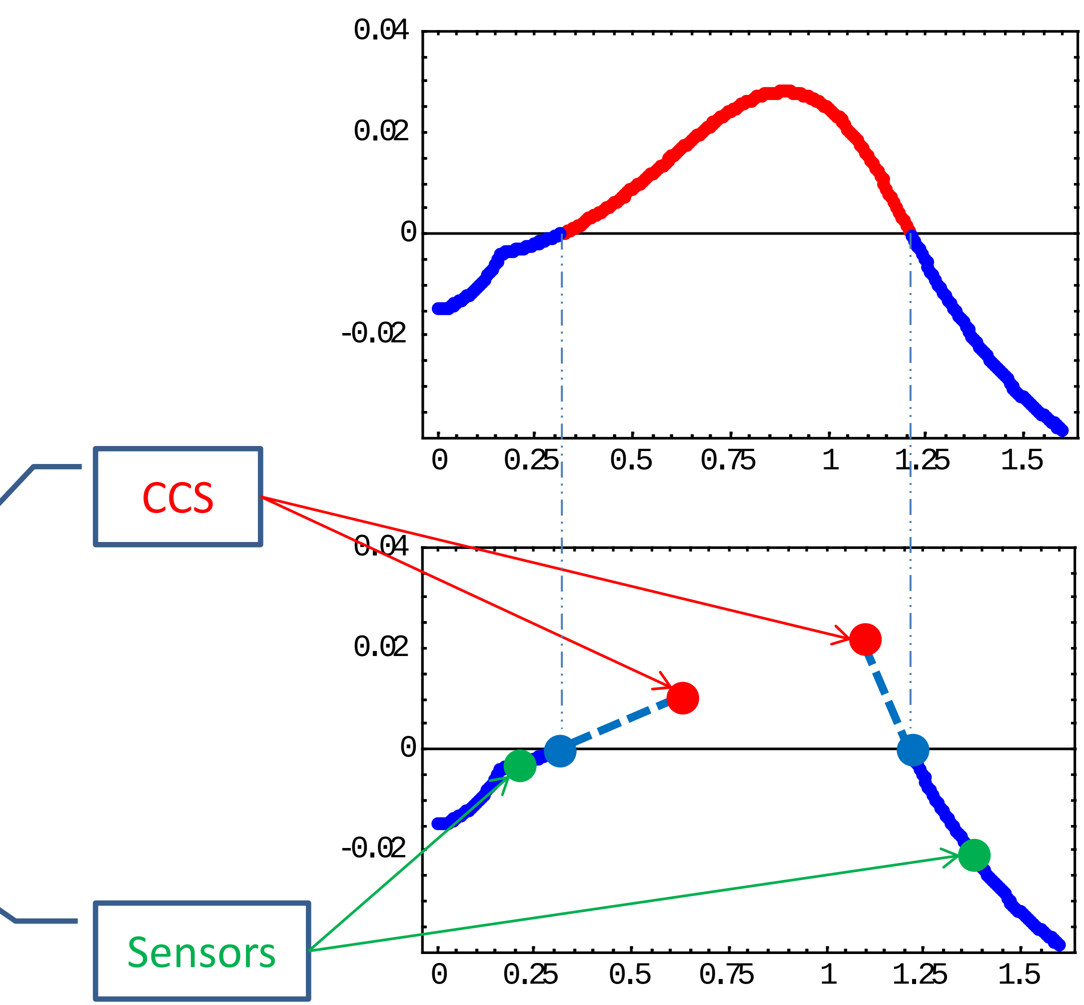
- CCS（コーシー条件面）法とは？
- ２種類の磁気センサーは必要か？

- RF電流駆動ダイバータプラズマ形状再構成
- SVD（特異値分解）、GCV（一般化交差検証法）

- ２種類の磁気計測によるダイバータプラズマ形状再構成

Principle of CCS method





Vacuum field is determined by boundary conditions. Value and slope on CCS are unknown, but those on sensor surface are known. After fitting, the plasma surface (shape) is reconstructed.

Principle on vacuum magnetic field

Maxwell equation (not equilibrium equation):

$$\nabla \times B = \mu_0 j \quad B = \nabla \times A \quad \phi = rA_\theta \rightarrow \nabla . \left(\frac{\nabla \phi}{r^2} \right) = -\frac{\mu_0 j}{r}$$

Boundary integral equation:

$$\sigma \cdot \phi(\vec{x}) + \int_{\partial\Omega} [G(\vec{x},\vec{y}) grad \phi(\vec{y}) - \phi(\vec{y}) grad G(\vec{x},\vec{y})] \frac{dS(\vec{y})}{r^2}$$

$$= \int_{\Omega} \mu_0 j_c(\vec{y}) G(\vec{x},\vec{y}) \frac{dV(\vec{y})}{r_y}$$

G is a Green function:

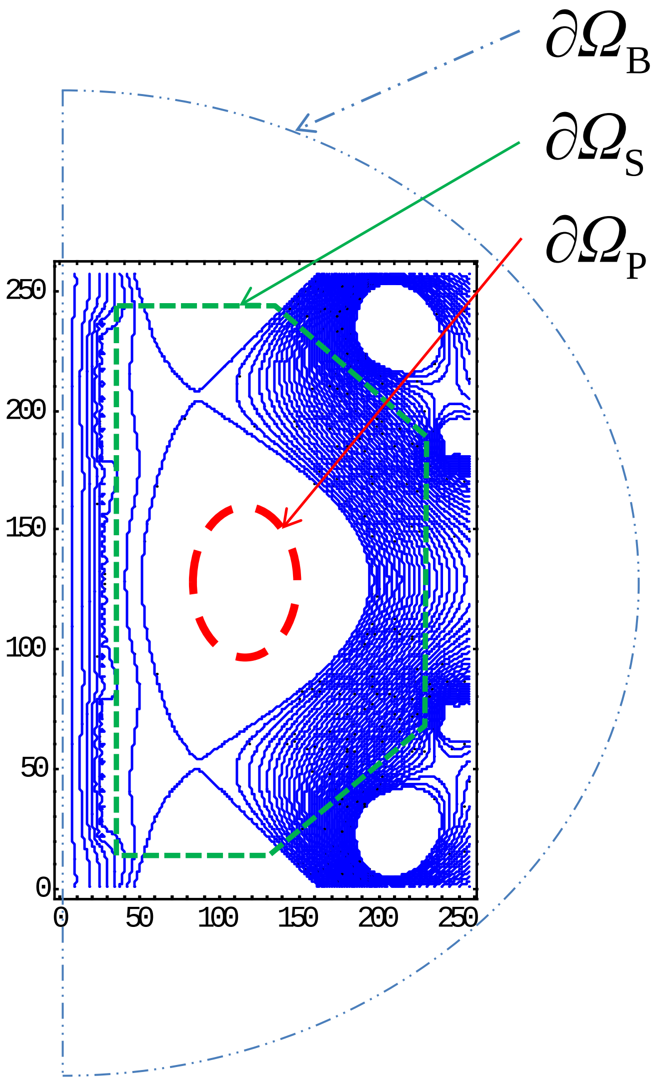
$$G(x,y) = G\left(r_x, z_x, r_y, z_y\right) = \frac{4}{k}\sqrt{r_x r_y}\left\{\left(1 - \frac{k^2}{2}\right)K(k) - E(k)\right\}$$

σ is a constant, defined as follows:

$$\sigma = \begin{cases} 8\pi^2 (\vec{x} \text{ in the } \Omega \text{ interior}) \\ 4\pi^2 (\vec{x} \text{ on } \partial\Omega) \\ 0 \quad (\vec{x} \text{ in the } \Omega \text{ exterior}) \end{cases}$$

References: [1] K. Kurihara: A new shape reproduction method based on the Cauchy-condition surface for real-time tokamak reactor control, Fusion Engineering and Design 51–52 (2000) 1049–1057.

CCS (Cauchy Condition Surface) method



Let's consider the area bounded by the boundary surface (B) and the hypothetical plasma surface (P).

$$\sigma \cdot \phi(\vec{x}) + \int_{\partial\Omega_P} [G(\vec{x},\vec{y}) grad\phi(\vec{y}) - \phi(\vec{y}) gradG(\vec{x},\vec{y})] \frac{dS(\vec{y})}{r^2}$$

$$= \int_{\Omega_{B-P}} \mu_0 j_c(\vec{y}) G(\vec{x},\vec{y}) \frac{dV(\vec{y})}{r_y}$$

A vacuum flux function value can be calculated at any point *x* outside the (P), if flux value and magnetic field on the (P).

Discretized equations of integral equations

(1) Flux value at flux loop on magnetic sensor surface

$$\boxed{\phi(\vec{x}_f)} = \sum_{i=1}^{M} W_{F1}(\vec{x}_f, \vec{z}_i)\boxed{\phi(\vec{z}_i)} + \sum_{i=1}^{M} W_{B1}(\vec{x}_f, \vec{z}_i)\boxed{Bt(\vec{z}_i)} + \sum_{i=1}^{NE} W_{E1}(\vec{x}_f, \vec{y}_i)\boxed{E(\vec{y}_i)} + \sum_{j=1}^{NC} W_{C1}(\vec{x}_f, \vec{y}_j)\boxed{I_{PF}(\vec{y}_j)}$$

Flux loop

Flux on CCS (unknown)

Magnetic field on CCS (unknown)

Eddy current (unknown)

PF coil current (measured)

(2) Magnetic field at magnetic probe on magnetic sensor surface

$$\boxed{Bt(\vec{x}_B)} = \sum_{i=1}^{M} W_{F2}(\vec{x}_B, \vec{z}_i)\boxed{\phi(\vec{z}_i)} + \sum_{i=1}^{M} W_{B2}(x_B, \vec{z}_i)\boxed{Bt(\vec{z}_i)} + \sum_{i=1}^{NE} W_{E2}(\vec{x}_B, \vec{y}_i)\boxed{E(\vec{y}_i)} + \sum_{j=1}^{NC} W_{C2}(\vec{x}_B, \vec{y}_j)\boxed{I_{PF}(\vec{y}_j)}$$

Magnetic probe

(3) Flux value at point on CCS

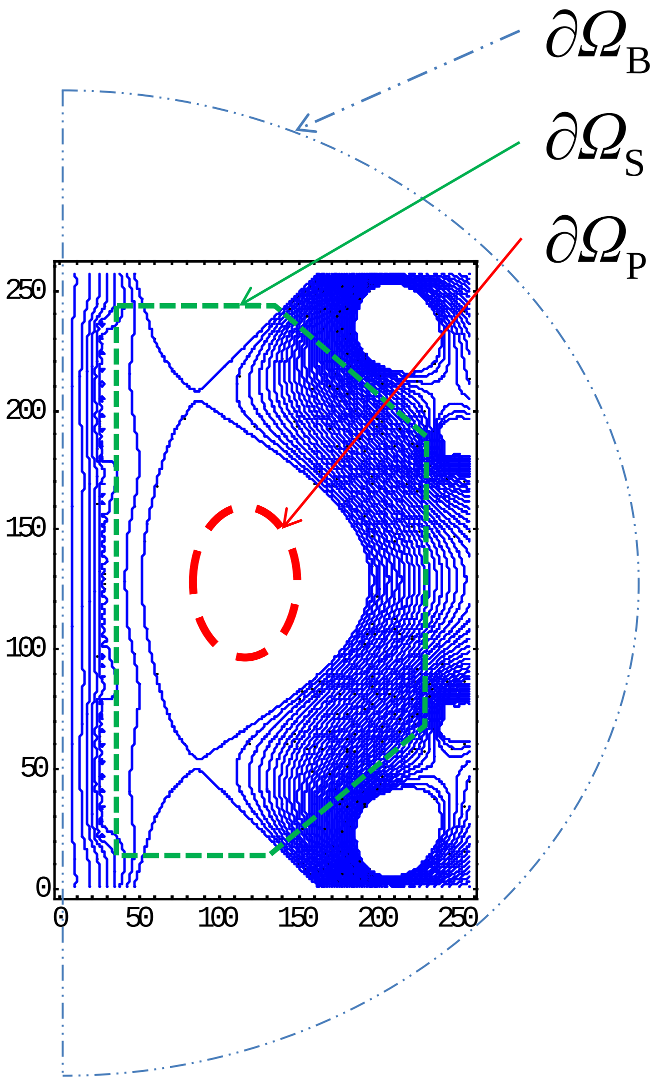
$$\boxed{\frac{1}{2}\phi(\vec{x}_C)} = \sum_{i=1}^{M} W_{F3}(\vec{x}_C, \vec{z}_i)\boxed{\phi(\vec{z}_i)} + \sum_{i=1}^{M} W_{B3}(\vec{x}_C, \vec{z}_i)\boxed{Bt(\vec{z}_i)} + \sum_{i=1}^{NE} W_{E3}(\vec{x}_C, \vec{y}_i)\boxed{E(\vec{y}_i)} + \sum_{j=1}^{NC} W_{C3}(\vec{x}_C, \vec{y}_j)\boxed{I_{PF}(\vec{y}_j)}$$

Flux on CCS (unknown)

Flux on CCS (unknown)

Magnetic field on CCS (unknown but deduced by this relation)

CCS (Cauchy Condition Surface) method



Let's consider the area bounded by the boundary surface (B) and the magnetic sensor surface (S).

$$\sigma \cdot \phi(\vec{x}) + \int_{\partial\Omega_S} [G(\vec{x},\vec{y}) grad\phi(\vec{y}) - \phi(\vec{y}) gradG(\vec{x},\vec{y})] \frac{dS(\vec{y})}{r^2}$$

$$= \int_{\Omega_{B-S}} \mu_0 j_c(\vec{y}) G(\vec{x},\vec{y}) \frac{dV(\vec{y})}{r_y}$$

A vacuum flux function value can be calculated at any point *x* outside the (P), if flux value and magnetic field on the (P).

Discretized equations of integral equations

(0) Flux value at point on magnetic sensor surface

$$\boxed{\frac{1}{2}\phi\left(\vec{x}_f\right)} = \sum_{i=1}^{N} W_{F0}\left(\vec{x}_f, \vec{z}_i\right)\boxed{\phi\left(\vec{z}_i\right)} + \sum_{i=1}^{NB} W_{B0}\left(\vec{x}_f, \vec{z}_i\right)\boxed{Bt\left(\vec{z}_i\right)} + \sum_{i=1}^{NE} W_{E1}\left(\vec{x}_f, \vec{y}_i\right)\boxed{E\left(\vec{y}_i\right)} + \sum_{j=1}^{NC} W_{C1}\left(\vec{x}_f, \vec{y}_j\right)\boxed{I_{PF}\left(\vec{y}_j\right)}$$

Flux loop
(known)

Flux loop
(known)

Magnetic probe
(magnetic field is deduced if eddy is known)
(if known, eddy is deduced by this equation)

Discretized equations of integral equations

- If eddy current does not exist, flux loop signal and magnetic probe signal are not independent.
- If eddy current does not exist, plasma shape is reconstructed uniquely from one kind of magnetic sensor.
- Even if another kind of magnetic sensor is added, the precision does not increase.
- As far as the number of magnetic sensors is increased, the precision increase depending on the number.
- If another kind of magnetic sensor is added, the eddy current is estimated.
- If another kind of magnetic sensor is added, plasma shape is reconstructed uniquely even if the eddy current exists.

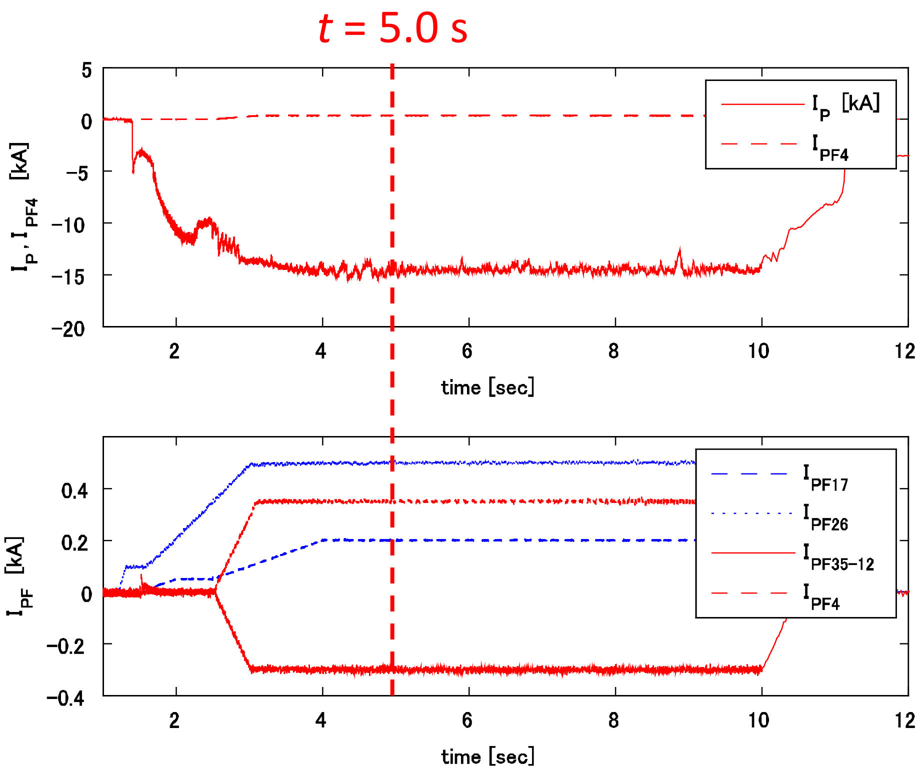
(0) Flux value at point on magnetic sensor surface

$$\boxed{\frac{1}{2}\phi(\vec{x}_f)} = \sum_{i=1}^{N} W_{F0}(\vec{x}_f, \vec{z}_i)\boxed{\phi(\vec{z}_i)} + \sum_{i=1}^{NB} W_{B0}(\vec{x}_f, \vec{z}_i)\boxed{Bt(\vec{z}_i)} + \sum_{i=1}^{NE} W_{E1}(\vec{x}_f, \vec{y}_i)\boxed{E(\vec{y}_i)} + \sum_{j=1}^{NC} W_{C1}(\vec{x}_f, \vec{y}_j)\boxed{I_{PF}(\vec{y}_j)}$$

Flux loop (known)

Flux loop (known)

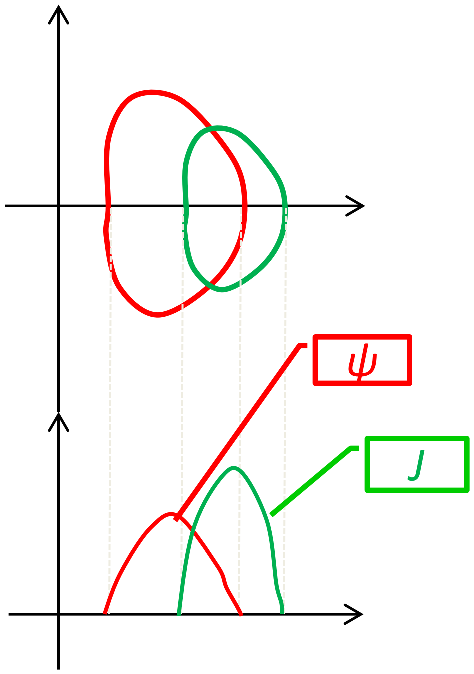
Magnetic probe (magnetic field is deduced if eddy is known) (if known, eddy is deduced by this equation)



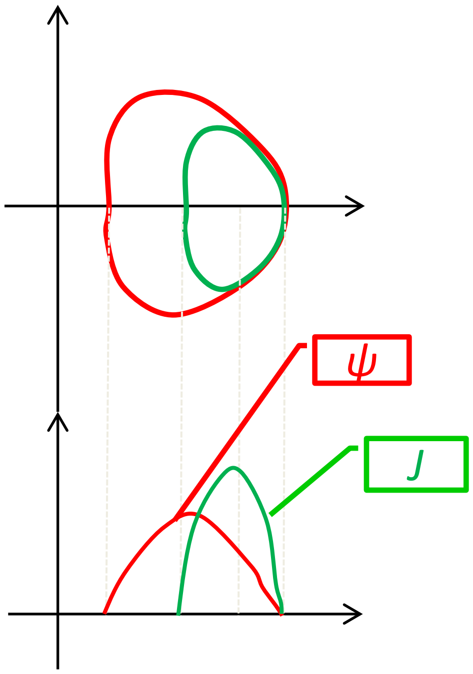
Shot No. 16499

Plasma is initiated by RF (8.2 GHz), and the plasma current is ramped up and sustained by RF (8.2 GHz).

Bt = 0.125 T @ R = 0.68 m



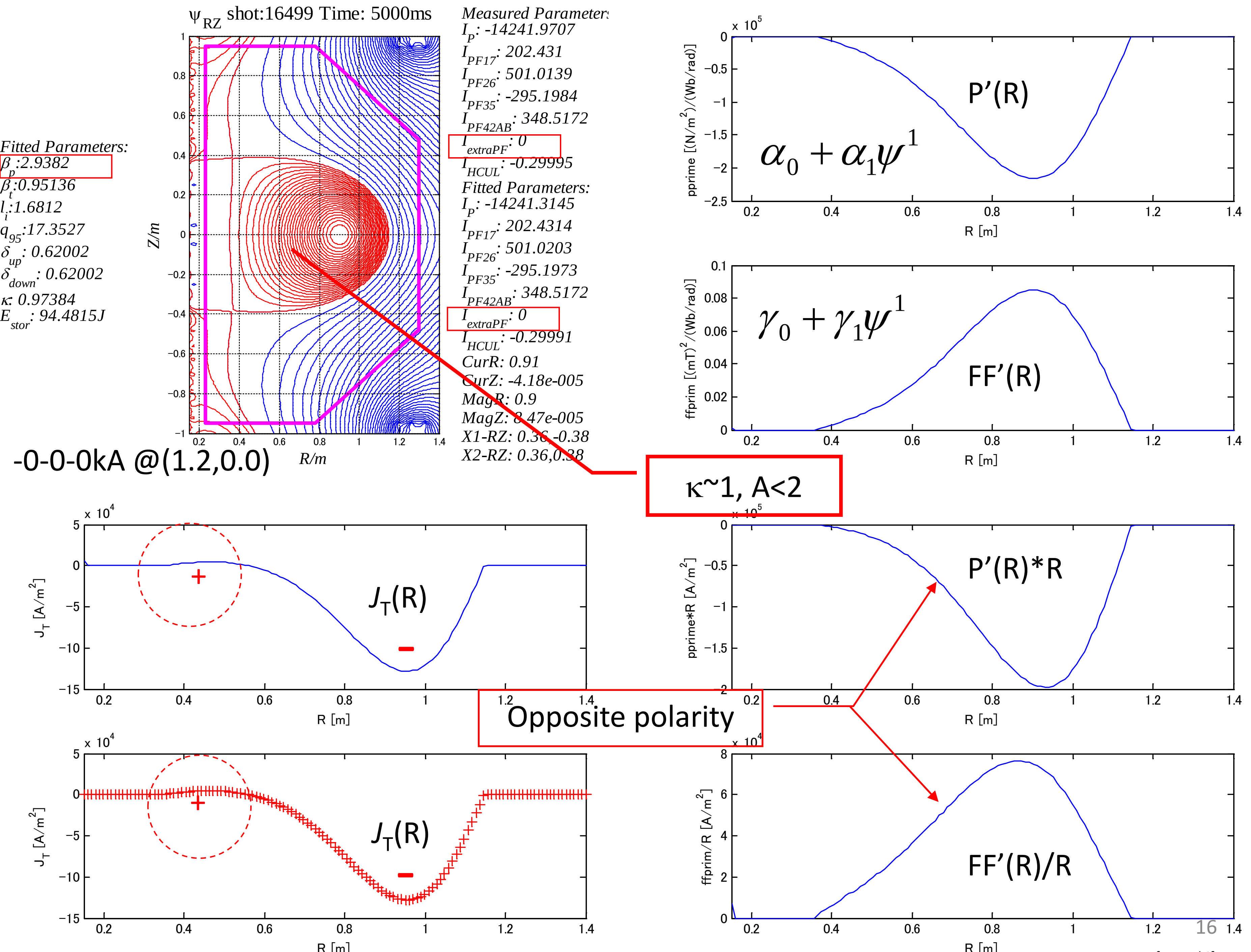
PCF (Plasma Current Fitting) method



EFIT code

Current density profile is adjusted so that the relevant magnetic flux and field are fitted to magnetic sensor signal.

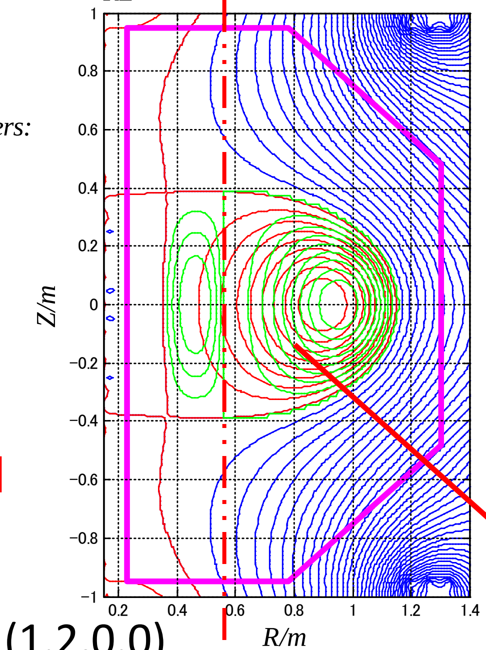
Current density profile is adjusted so that the relevant magnetic flux and field are fitted to magnetic sensor signal. The current density profile is constructed so as to satisfy plasma equilibrium with isotropic pressure.



After Ip drift correction

J_{RZ} shot:16499 Time: 5000ms

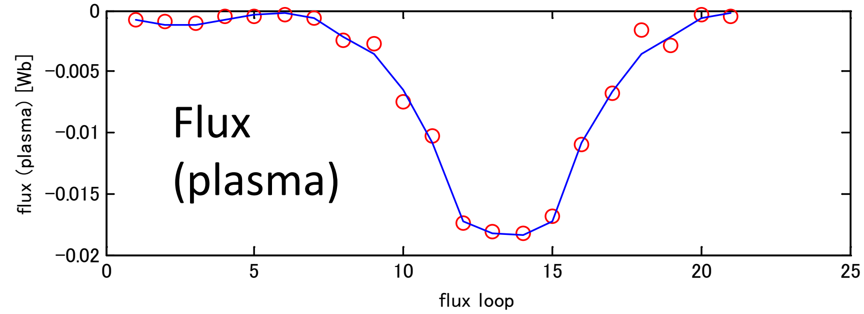
Fitted Parameters:
β_p:2.9382
β_t:0.95136
l_i:1.6812
q_{95}:17.3527
δ_{up}: 0.62002
δ_{down}: 0.62002
κ: 0.97384
E_{stor}: 94.4815J
I_P: -14536.9 A
I_P^+: 295.5822 A

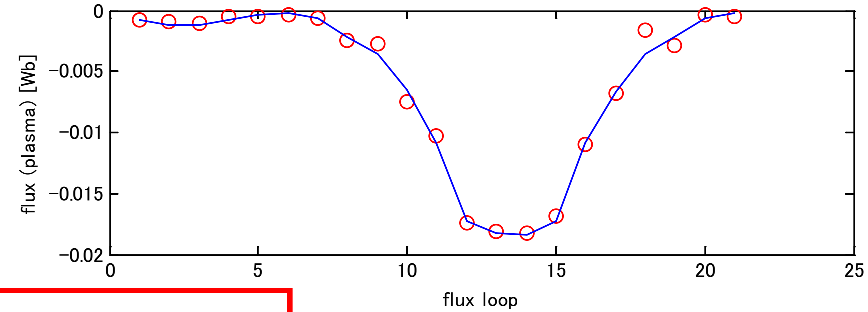


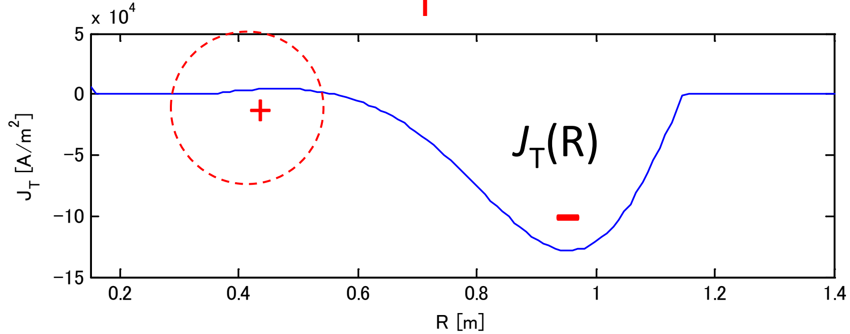
I_P: -14241.9707
I_{PF17}: 202.431
I_{PF26}: 501.0139
I_{PF35}: -295.1984
I_{PF42AB}: 348.5172
$I_{extraPF}$: 0
I_{HCUL}: -0.29995
Fitted Parameters:
I_P: -14241.3145
I_{PF17}: 202.4314
I_{PF26}: 501.0203
I_{PF35}: -295.1973
I_{PF42AB}: 348.5172
$I_{extraPF}$: 0
I_{HCUL}: -0.29991
CurR: 0.91
CurZ: -4.18e-005
MagR: 0.9
MagZ: 8.47e-005
X1-RZ: 0.36,-0.38
X2-RZ: 0.36,0.38

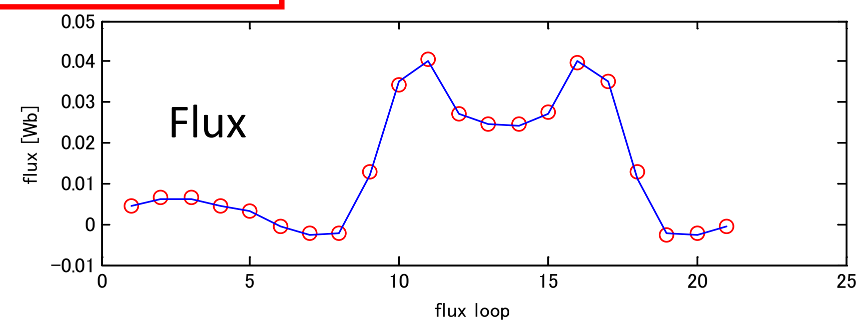
-0-0-0kA @(1.2,0.0)

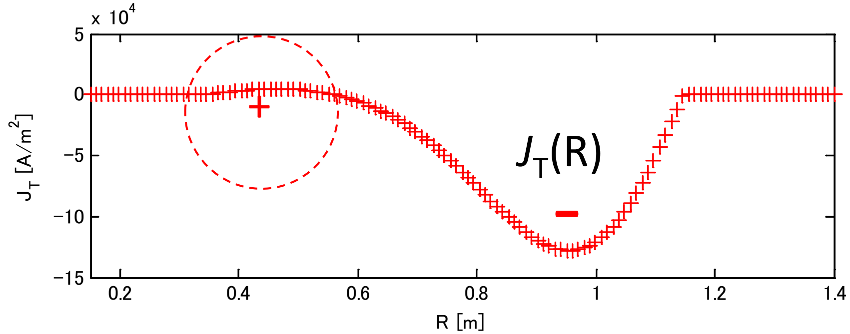
κ~1.4, A>2

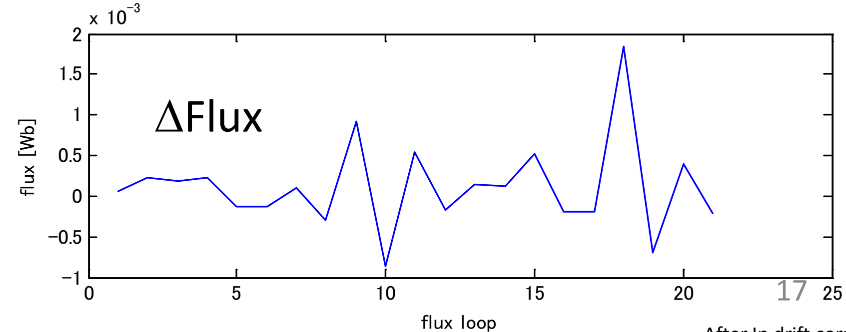












After Ip drift correction

Grad-Shafranov Equation

$$-\left(\nabla\times B\right)_T = \frac{1}{R}\left(R\frac{\partial}{\partial R}\frac{1}{R}\frac{\partial\psi}{\partial R}+\frac{\partial^2\psi}{\partial z^2}\right) = -\mu_0 J_T$$

$$F(\psi) = \mu_0 f(\psi) = RB_T$$

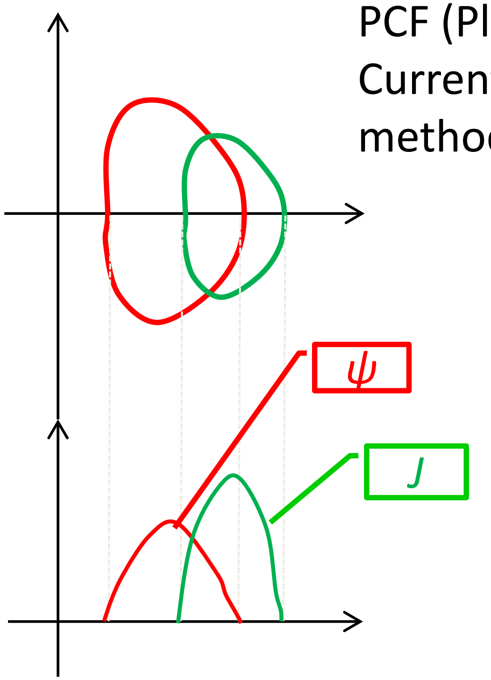
$$J_T[A/m^2] = P'(\psi)*R + FF'(\psi)/\left(\mu_0 R\right)$$

$$= J_0\left[\beta_J\frac{R}{R_0}g(\psi)+\left(1-\beta_J\right)\frac{R_0}{R}h(\psi)\right]$$

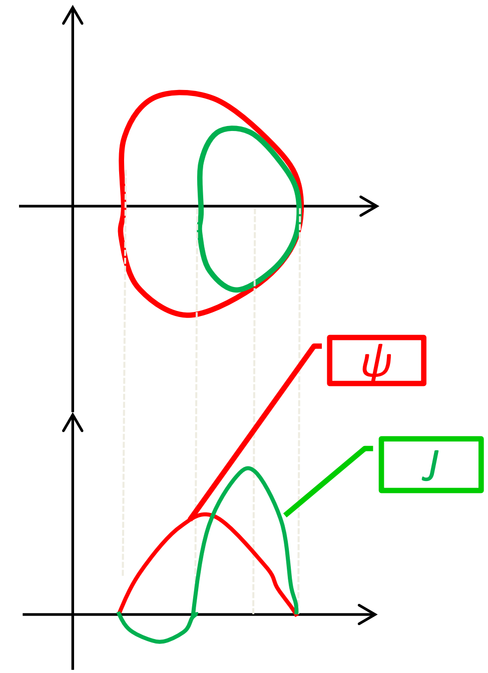
(For uniform current profile)

When $\beta_J > 1$, the first term and the second term have the opposite polarity.

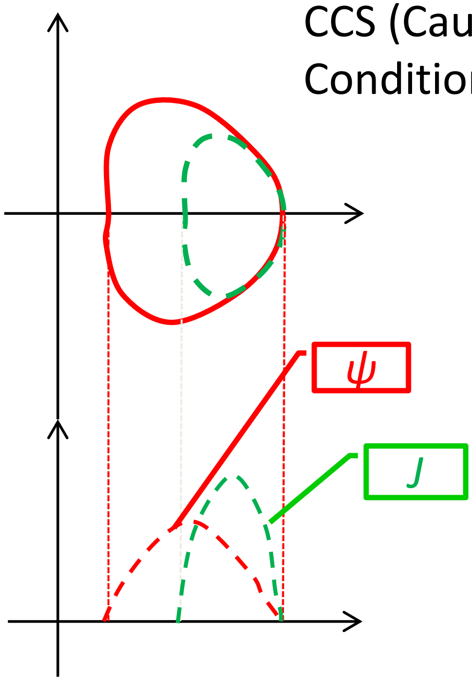
PCF (Plasma Current Fitting) method



EFIT code



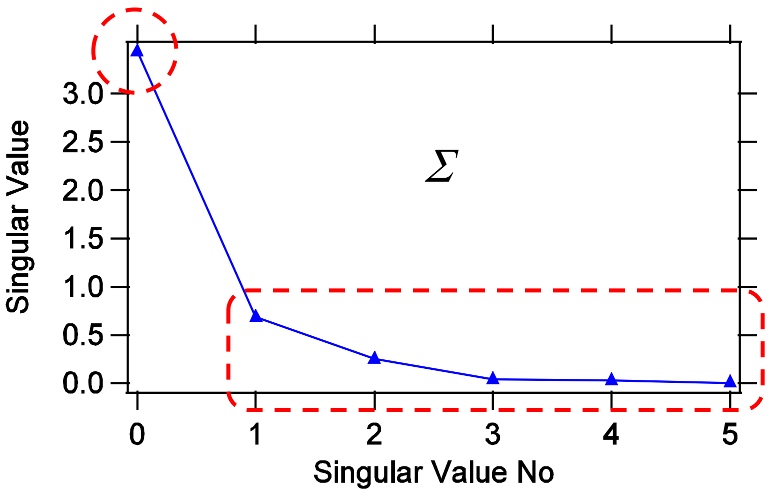
CCS (Cauchy Condition Surface)

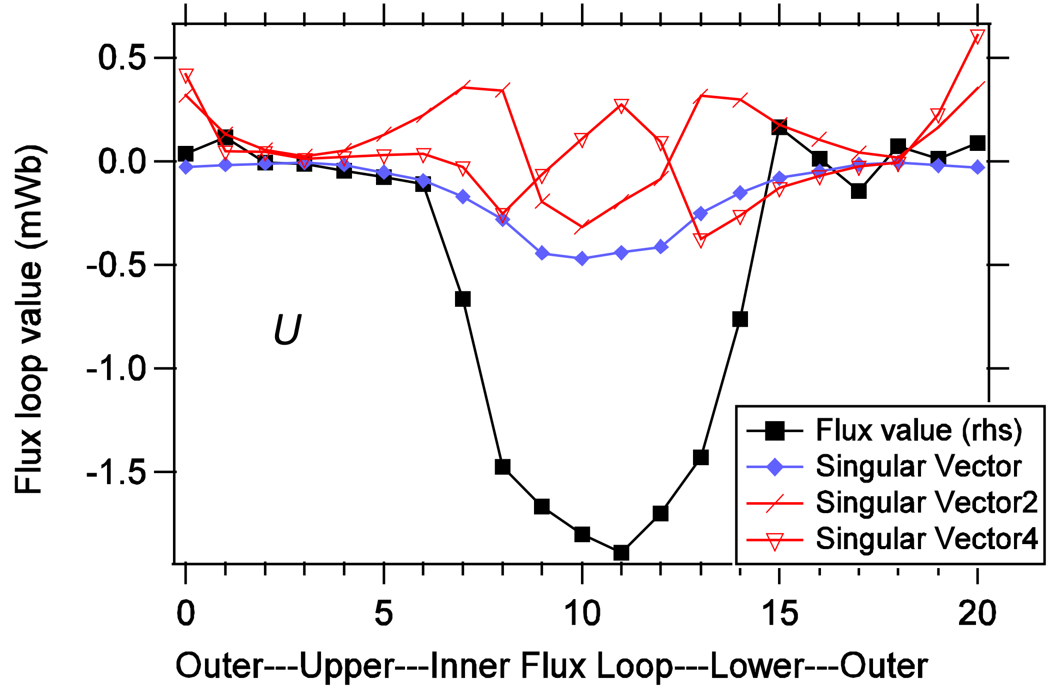


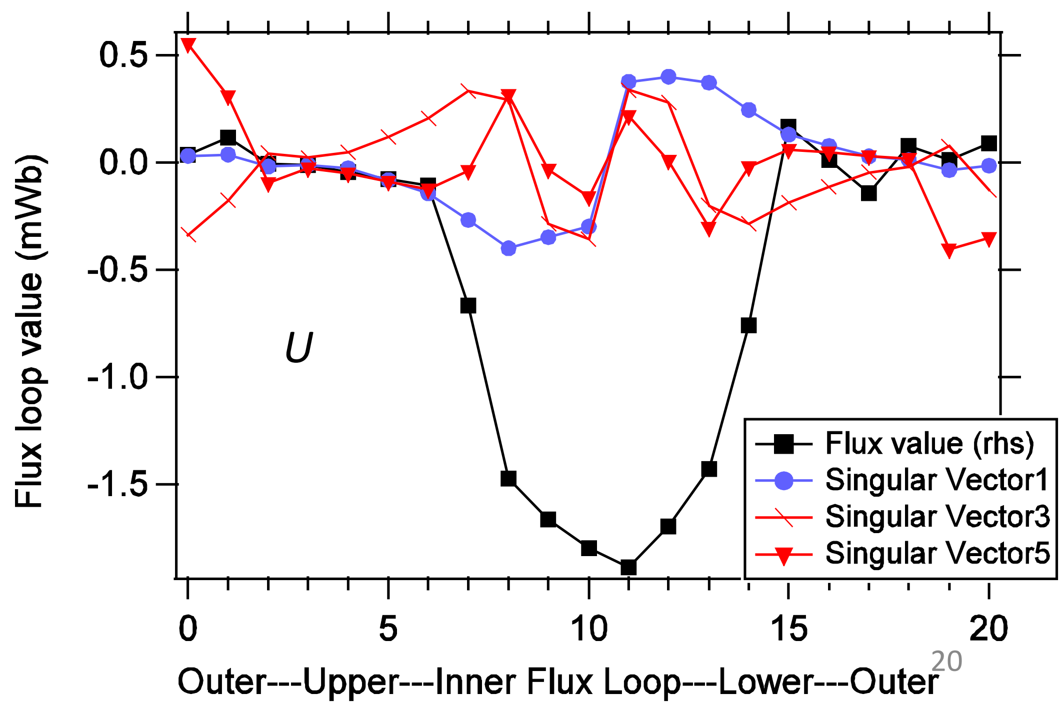
Observation equation: *Ax* = *b*

$$A = U\Sigma V^T$$

Normalized singular vectors







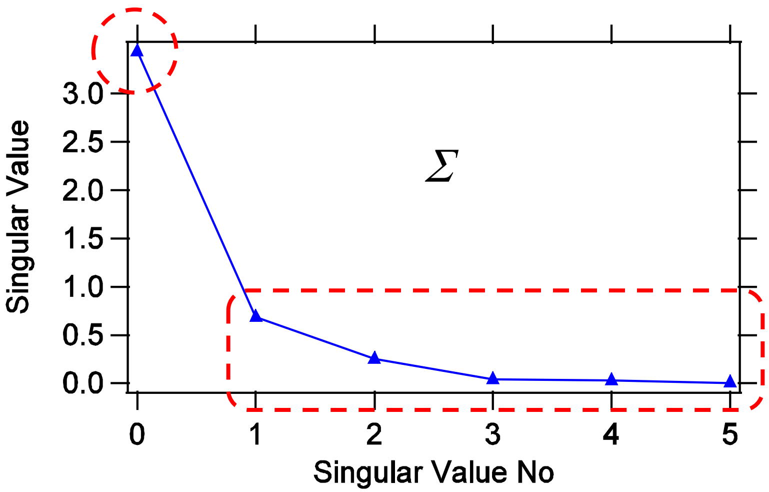
- The 0th singular value is much larger than the other singular values.
- The 0th, 2nd and 4th singular vectors are even with respect to the equatorial plane. The 1st, 3rd and 5th singular vectors are odd.
- Plasma shape may be reconstructed with the even singular vectors.
- Cumulative contribution ratio of the 0th SV is over 95 %.

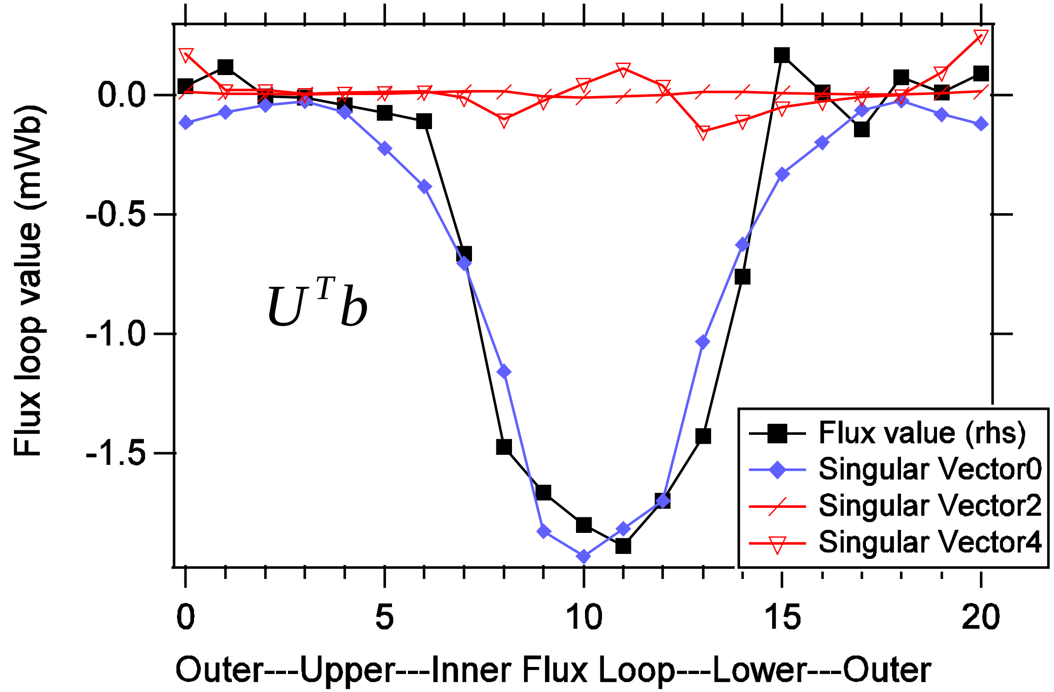
$$\frac{\sigma_0^2}{\sigma_0^2 + \sigma_1^2 + \ldots + \sigma_5^2}$$

Observation equation: $Ax = b$

$$A = U\Sigma V^T$$

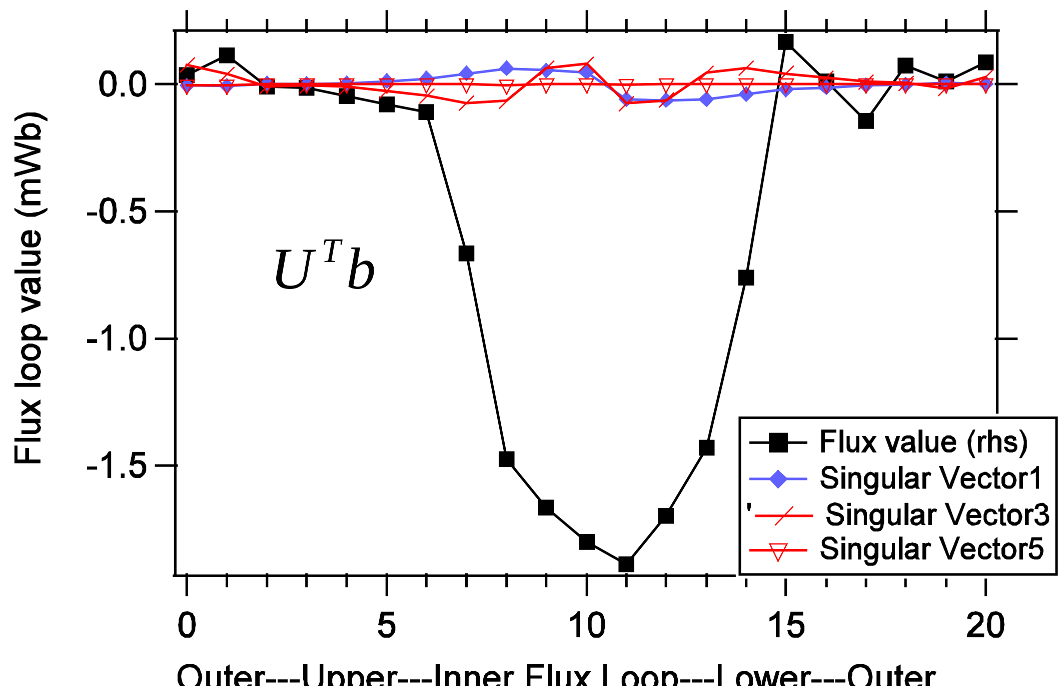
Fitted singular vectors



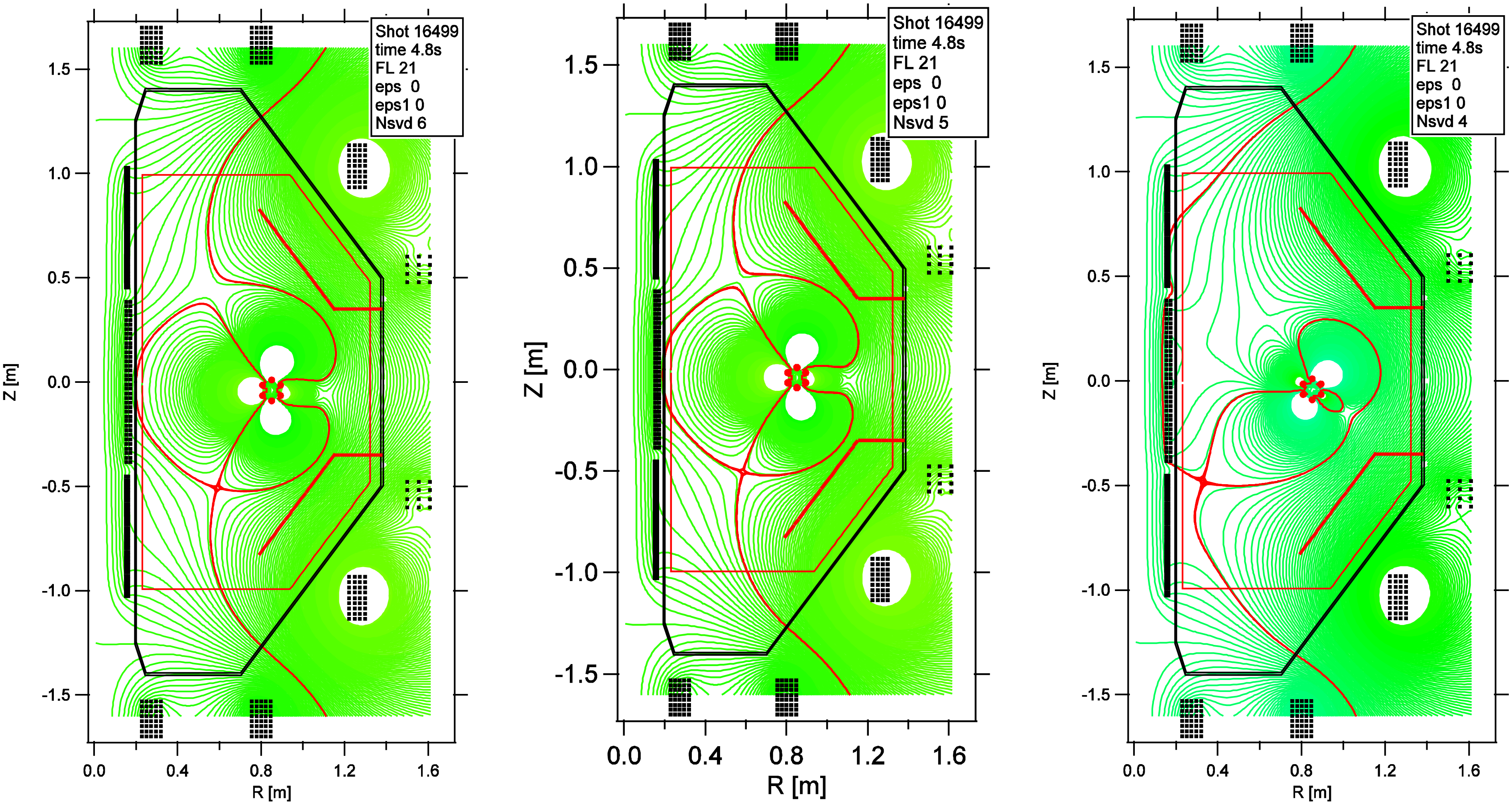


- The 0th singular value is much larger than the other singular values.
- The 0th singular component is much larger than the other singular components.
- Cumulative contribution ratio of the 0th SV is over 95 %.

$$\frac{\sigma_0^2}{\sigma_0^2 + \sigma_1^2 + \ldots + \sigma_5^2}$$

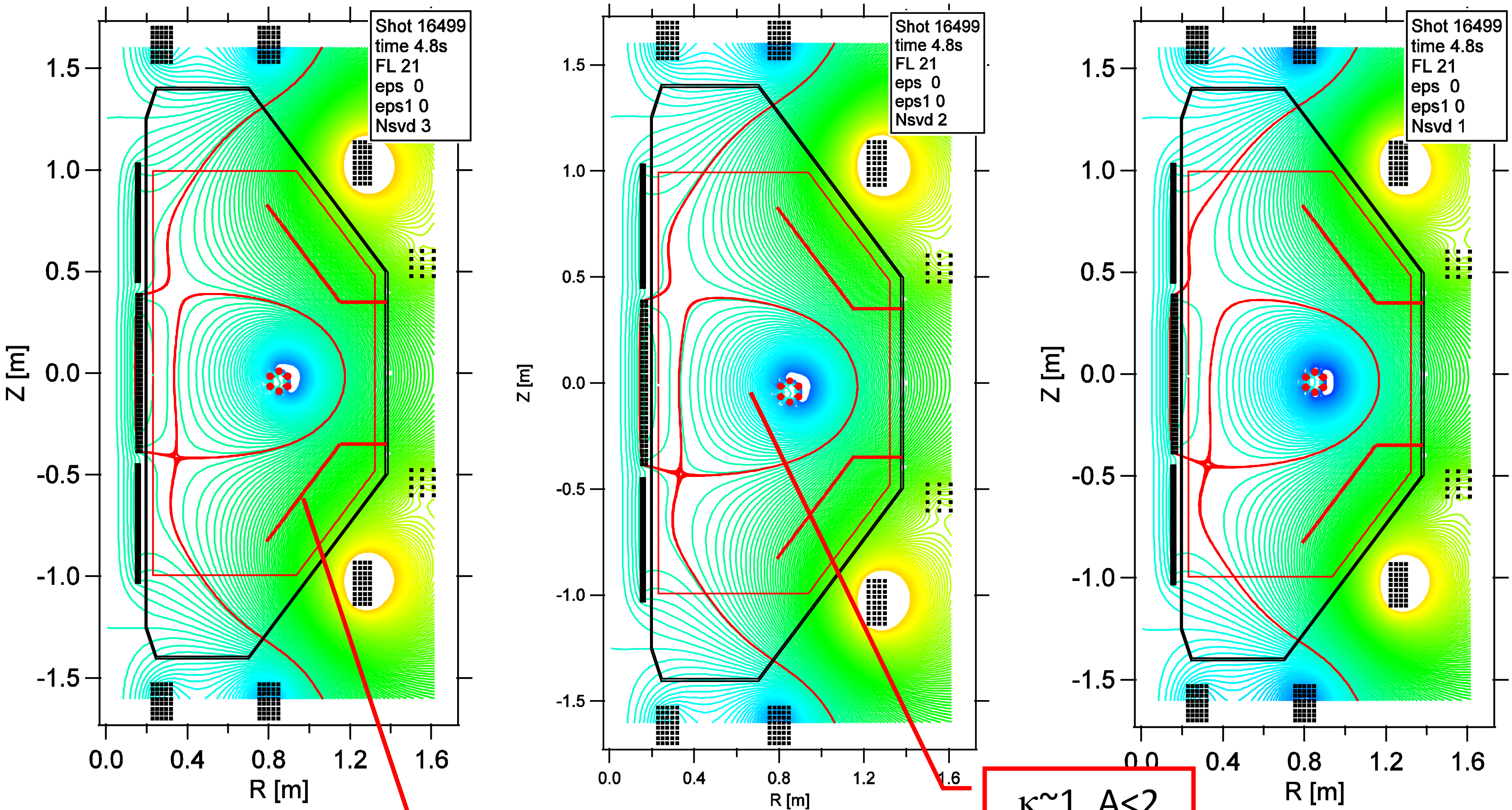


Truncation of small SV components



Tikhonov eps=0.000 CCSM=6 **Nsvd=6 (All components)** CCR = 100.0 %	Tikhonov eps=0.000 CCSM=6 **Nsvd=5** CCR > 99.99 %	Tikhonov eps=0.000 CCSM=6 **Nsvd=4** CCR > 99.99 %

Ill-posedness disappears for Nsvd smaller than 3



Tikhonov eps=0.000
CCSM=6
Nsvd=3
CCR > 99.9 %

Hot Wall

Tikhonov eps=0.000
CCSM=6
Nsvd=2
CCR = 99.5 %

Tikhonov eps=0.000
CCSM=6
Nsvd=1 (Only the 0th SV)
CCR = 95.6 %

How do we judge the truncation of small SV?

- Regularization of SVD
  - Truncation of small SV
  - Filtering of small SV

$$Ax = b,$$ Observation equation

$$A = \left(U\Sigma V^T\right),$$ The coefficient matrix is decomposed (SVD),

$$x = \left(V\Sigma^{-1}U^T\right)b$$ The solution is expressed by the SVD.

$$\Sigma = Diag\left(\sigma_1 \quad \sigma_2 \quad \dots \quad \sigma_6\right)$$

$$\Sigma^{-1} = Diag\left(\frac{1}{\sigma_1} \quad \frac{1}{\sigma_2} \quad \dots \quad \frac{1}{\sigma_6}\right)$$

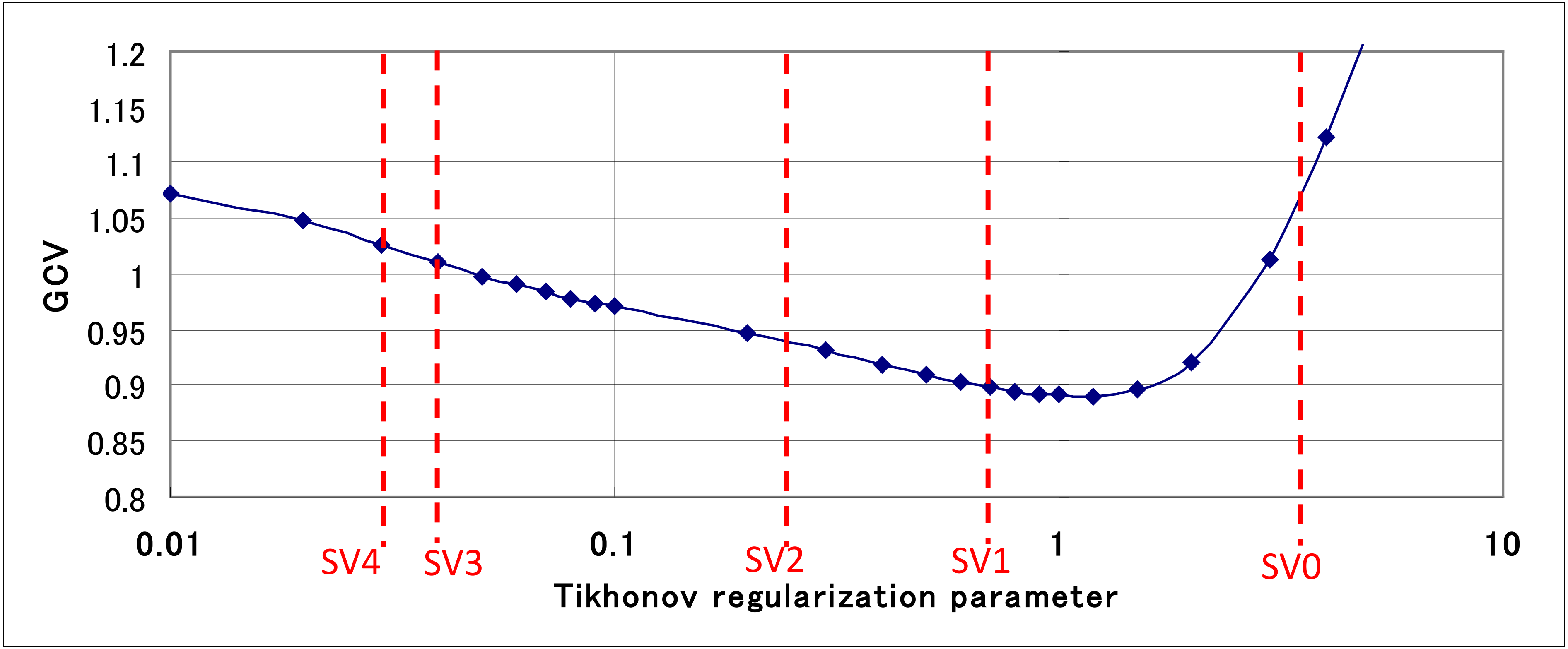
$Wiener\ \ Filter$ Is introduced to filter out the small SV components.

$$\Sigma^{-1} \Rightarrow Diag\left(\frac{1}{\sigma_1}\boxed{\frac{\sigma_1^2}{\sigma_1^2+\lambda^2}} \quad \frac{1}{\sigma_2}\boxed{\frac{\sigma_2^2}{\sigma_2^2+\lambda^2}} \quad \dots \quad \frac{1}{\sigma_6}\boxed{\frac{\sigma_6^2}{\sigma_6^2+\lambda^2}}\right)$$

Wiener Weight

How can we determine the filter parameter λ?

Optimal Criterion Function for Generalized Cross Validation

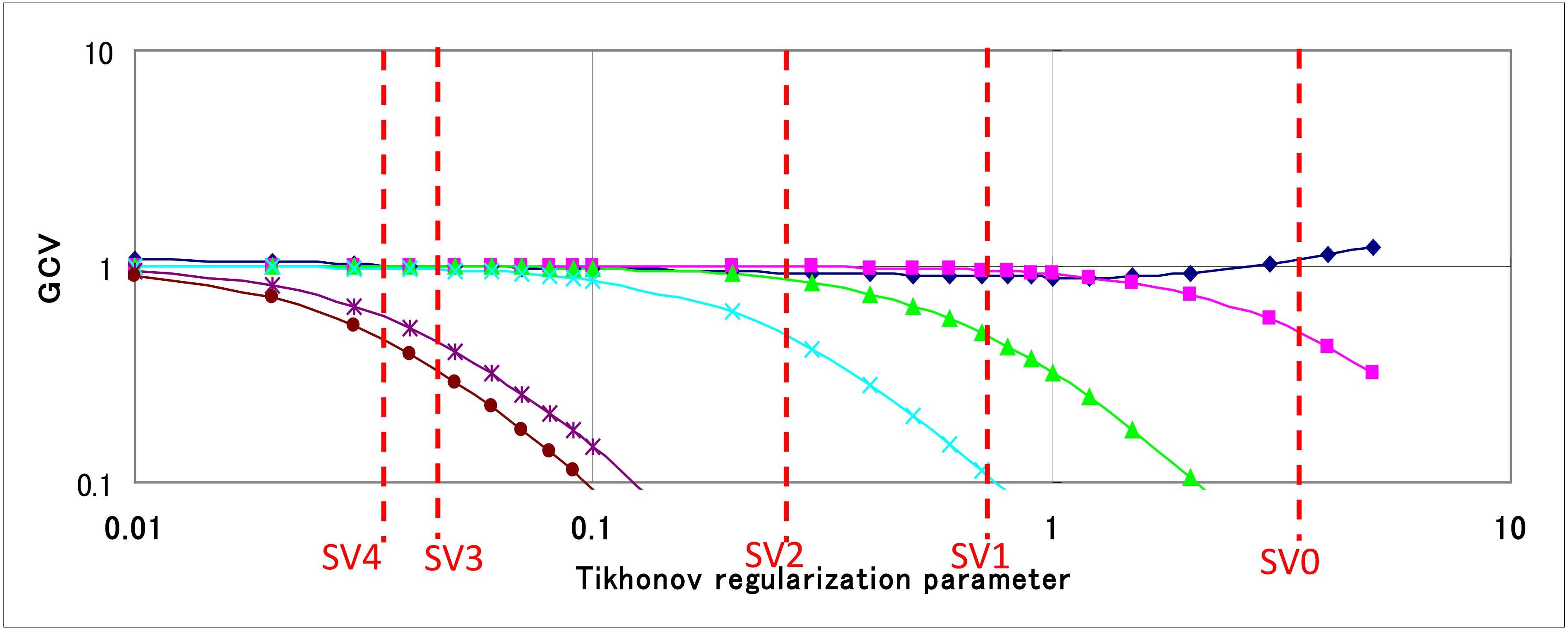


In GCV, we estimate the error by leaving one out and fitting. And minimize the weighted average of the squared error with respect to regularization parameter λ.

$$GCV = \frac{1}{n}\sum_{\alpha=1}^{n} w_\alpha \left(b_\alpha - A^{(-\alpha)}x_\alpha\right)^2 = \frac{\sum\left(\frac{\lambda^2}{\sigma^2+\lambda^2}c\right)^2 + \left\|\Delta b\right\|^2}{\left(\sum\frac{\lambda^2}{\sigma^2+\lambda^2} + (n-m)\right)^2}$$

$$Ax = b, \quad A = U\Sigma V^T, \quad \Delta b = b - UU^T b, \quad c = U^T b$$

Optimal Criterion Function for Generalized Cross Validation

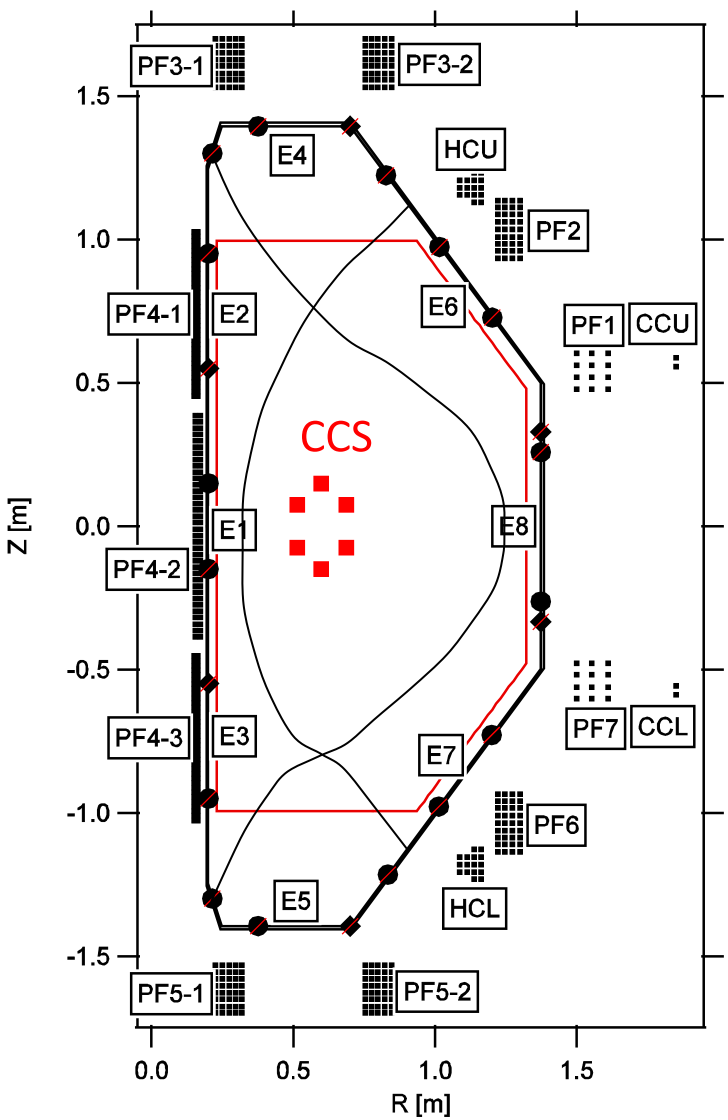


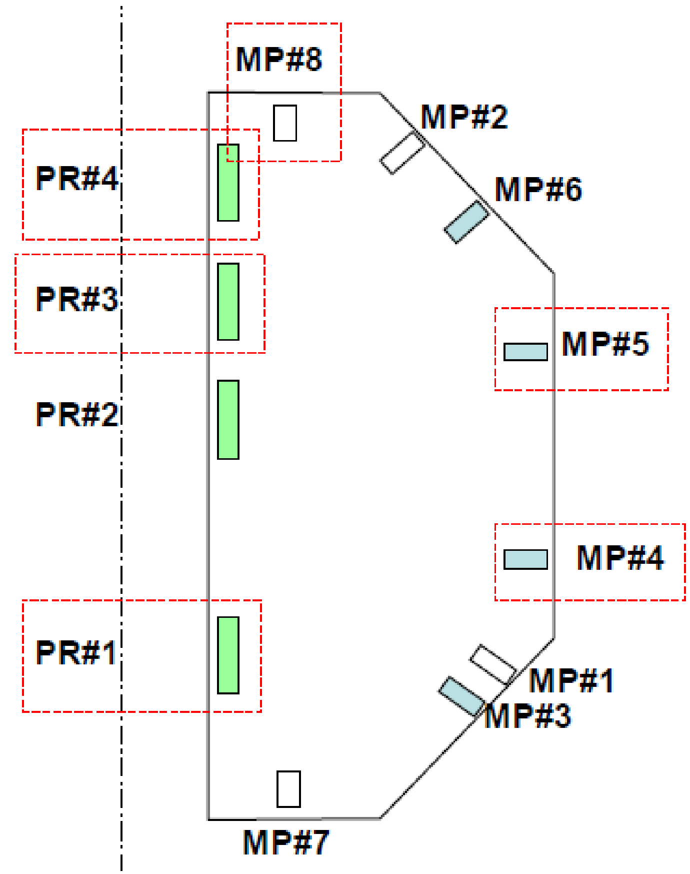
In GCV, we estimate the error by leaving one out and fitting. And minimize the weighted average of the squared error with respect to regularization parameter λ.

$$GCV = \frac{1}{n}\sum_{\alpha=1}^{n} w_\alpha \left(b_\alpha - A^{(-\alpha)} x_\alpha\right)^2 = \frac{\sum \left(\frac{\lambda^2}{\sigma^2+\lambda^2} c\right)^2 + \left\|\Delta b\right\|^2}{\left(\sum \frac{\lambda^2}{\sigma^2+\lambda^2} + (n-m)\right)^2}$$

$$Ax = b, \quad A = U\Sigma V^T, \quad \Delta b = b - UU^T b, \quad c = U^T b$$

Installing magnetic probes





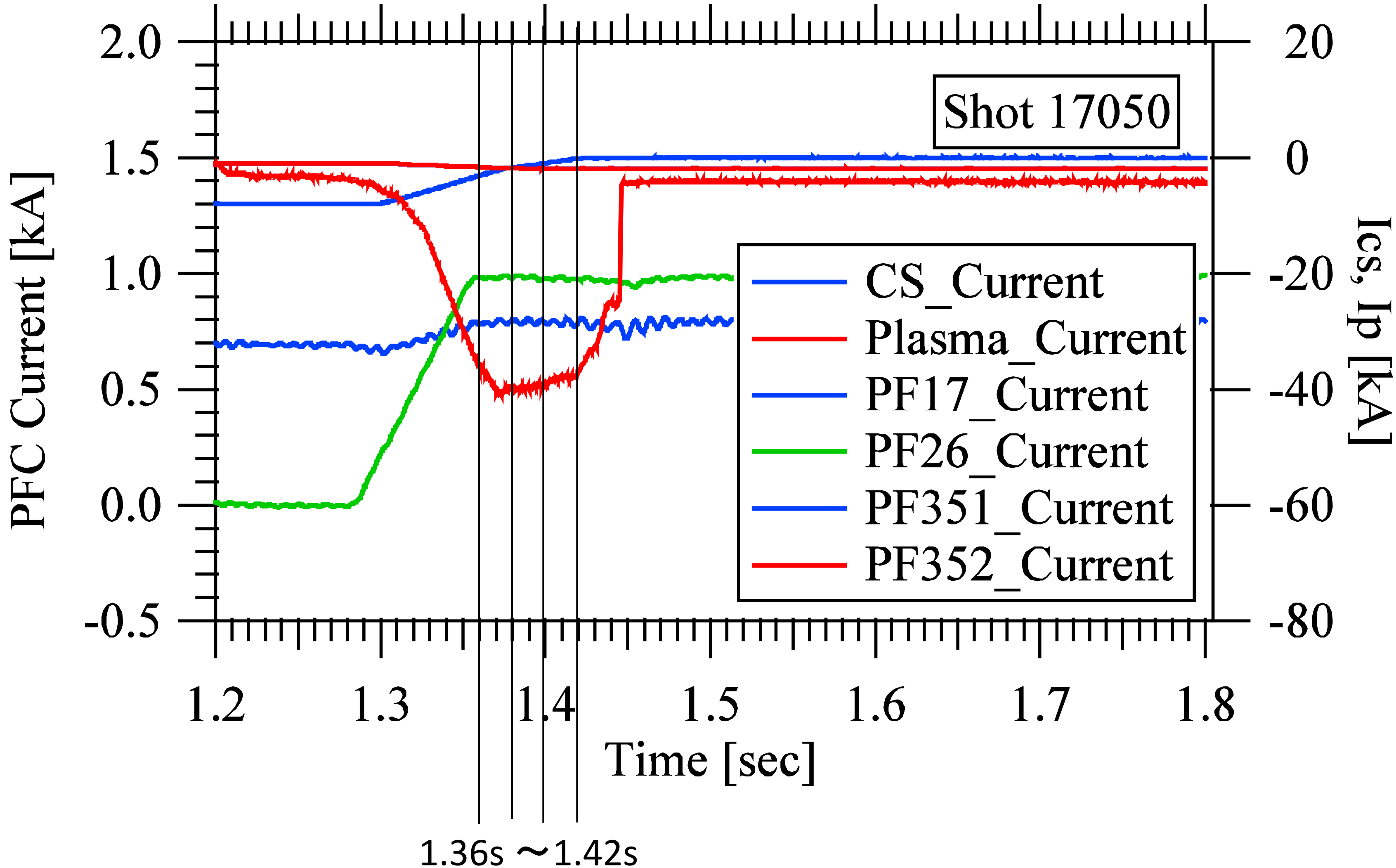
MP#1,2,7,8: Single heat-resistant MP

MP#3,4,5,6: Train of heat-resistant MP

PR#1,2,3,4: Long MP wound with MI cable

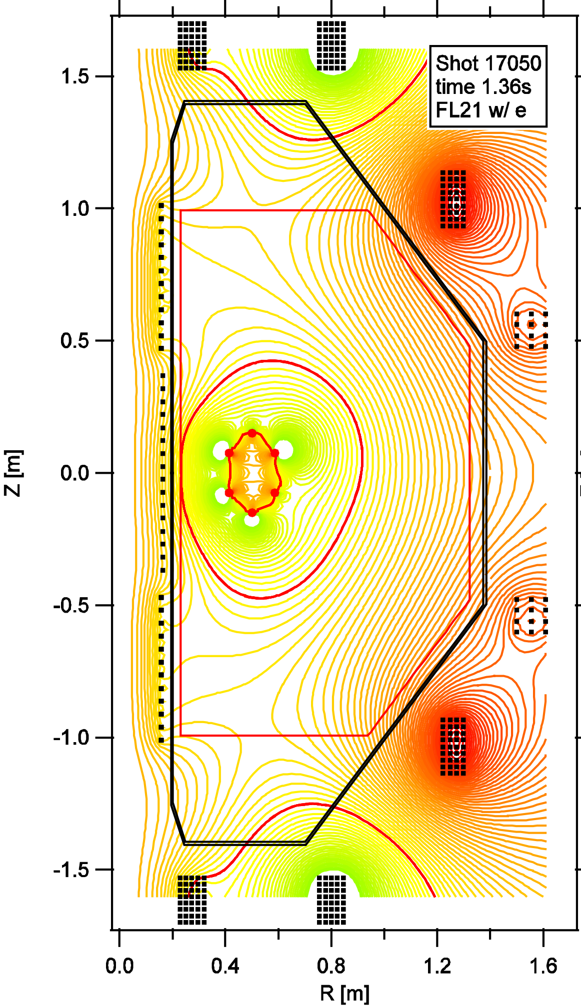
: Wiring disconnection

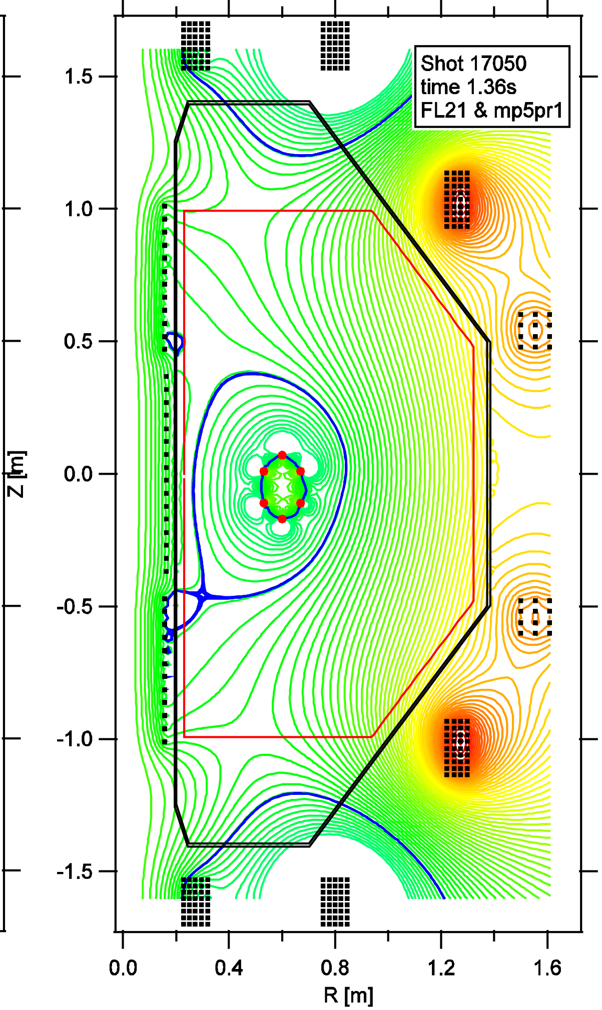
Shape reconstruction with 21 flux loops, 5 magnetic probes and 1 partial rogowski

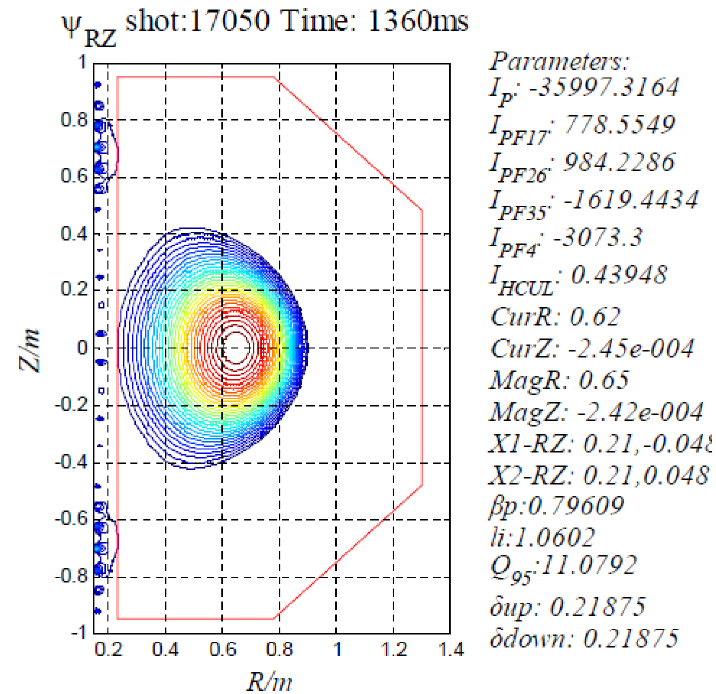


Flux loop averages the signal toroidally along the flux loop.

Circular







E-FIT

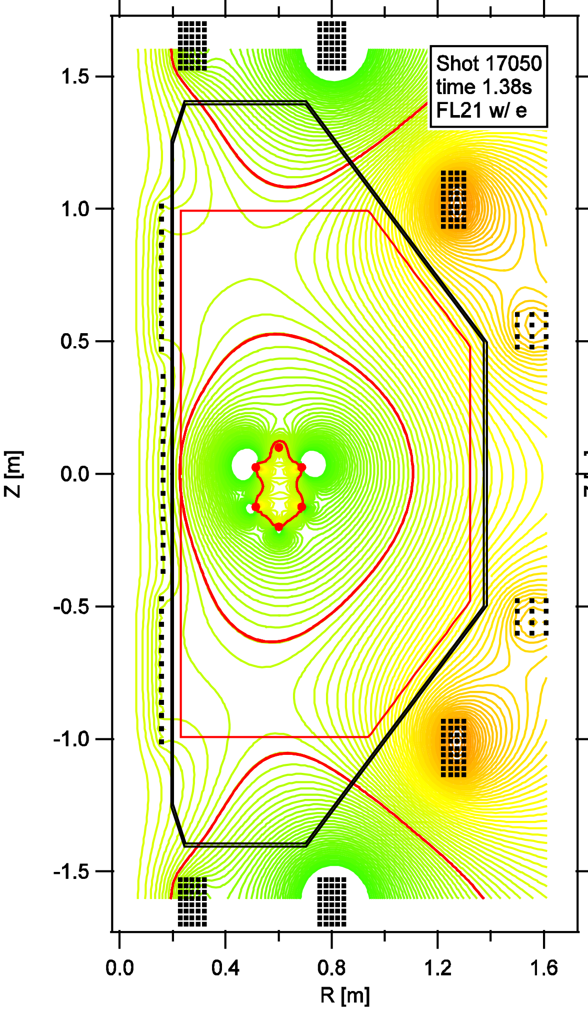
21 FL

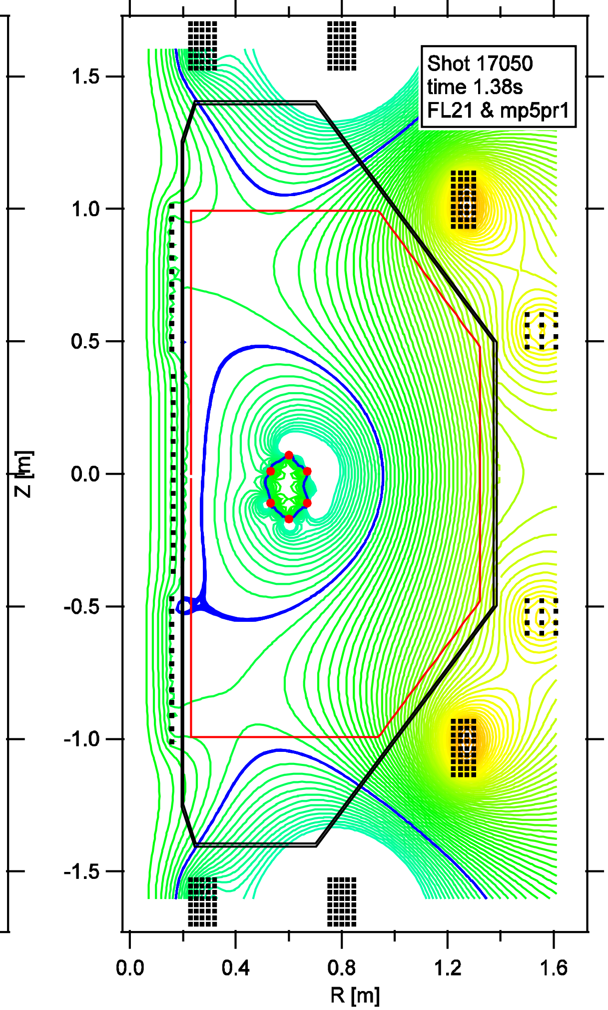
Circular plasma is obtained. Can be reconstructed even without eddy current.

21FL+5MP+1PR

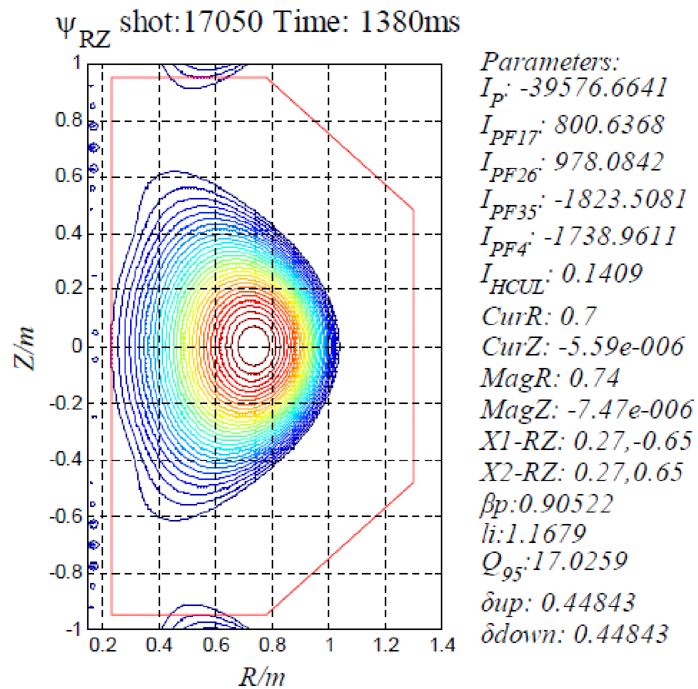
Strong effect from inner PR remains.

When magnetic probes are added, plasma shape can be reconstructed with eddy current and sigma BT adjustment.





Elongated



Ipf35-12 reaches maximum at t=1.38s.

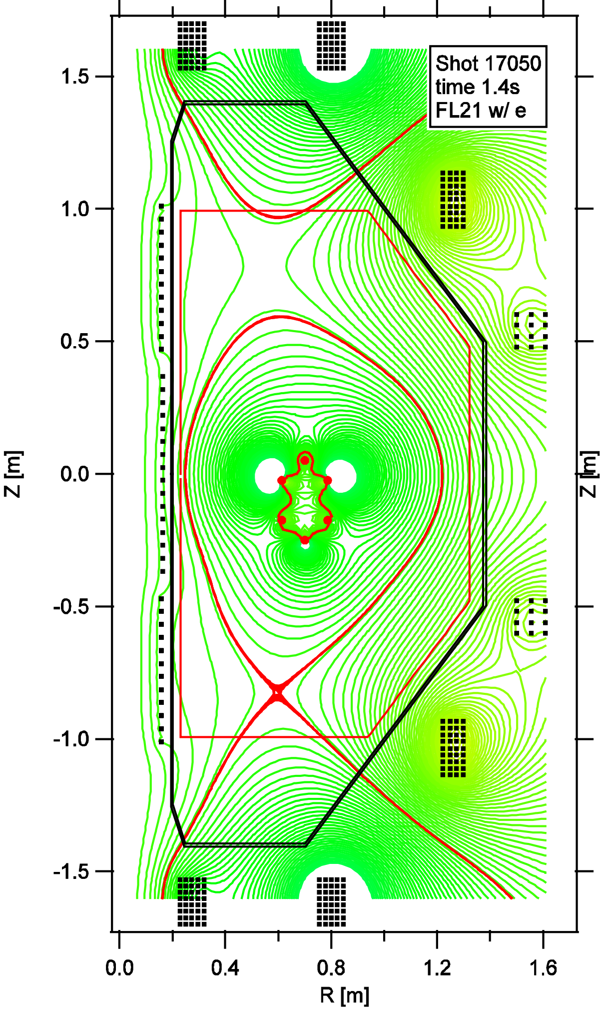
21 FL

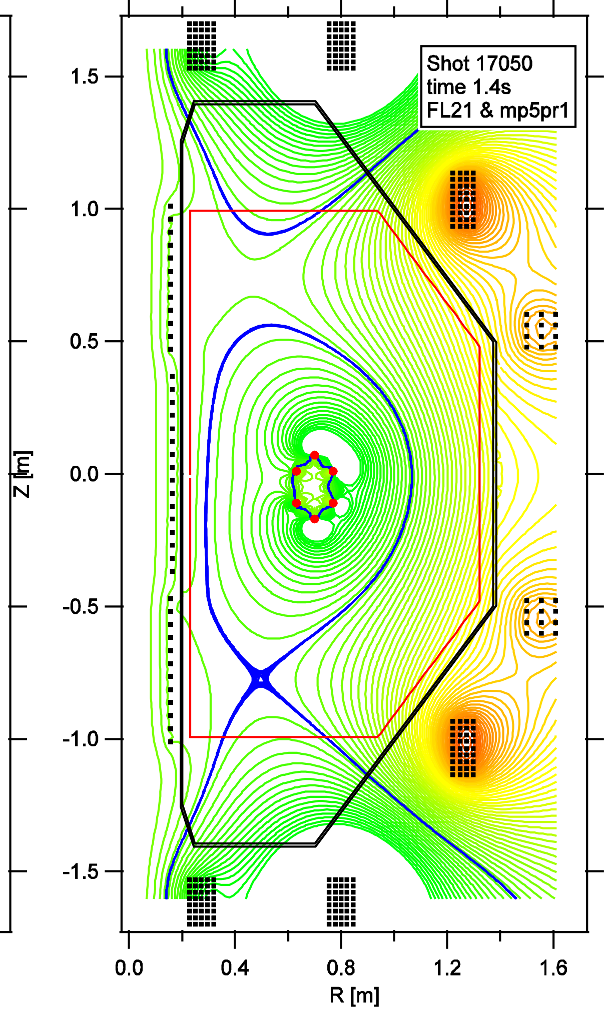
Elongated plasma is obtained. Can be reconstructed even without eddy current.

21FL+5MP+1PR

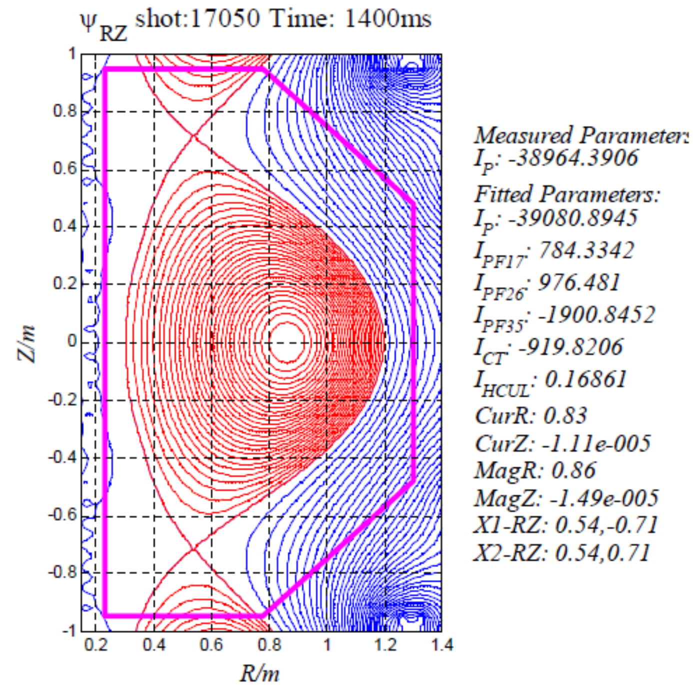
Strong effect from inner PR remains.

When magnetic probes are added, plasma shape can be reconstructed with eddy current and sigma BT adjustment.





X-point



Ipf4 approaches zero toward t=1.42s.

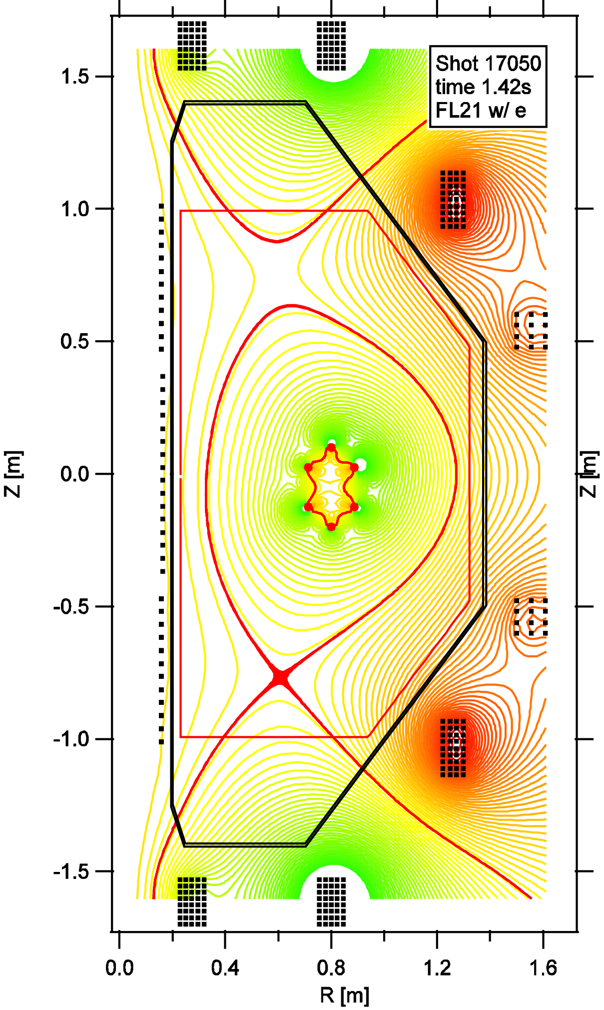
21 FL

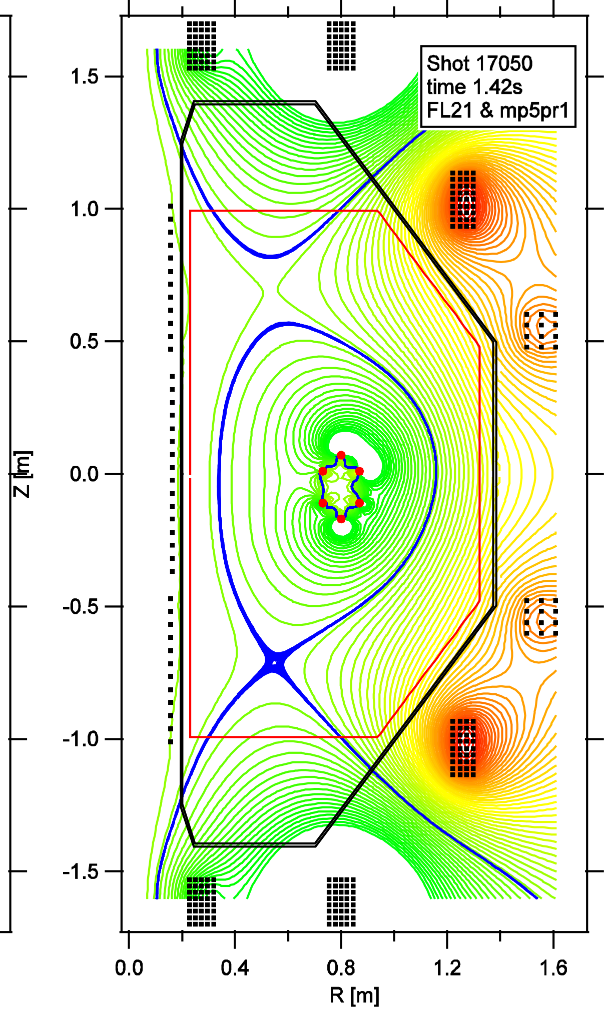
X-point is obtained. Can be reconstructed even without eddy current.

21FL+5MP+1PR

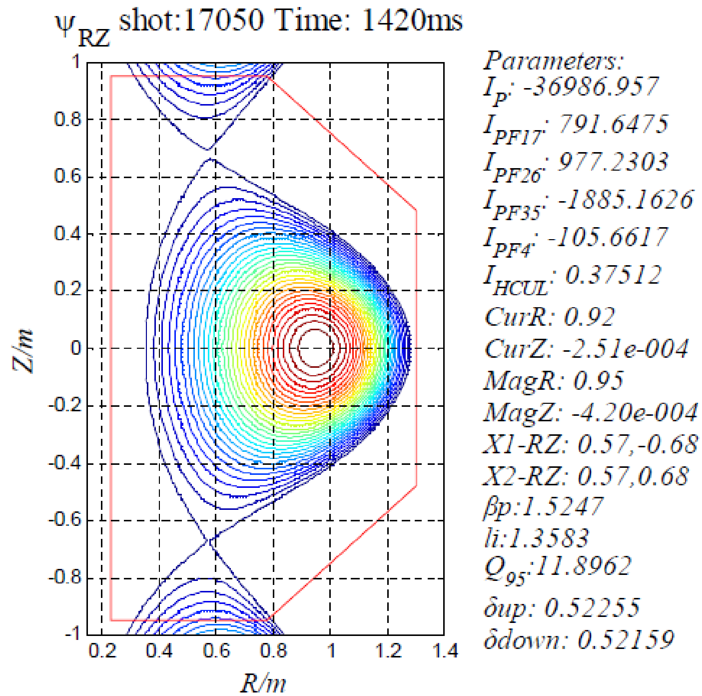
A little effect from inner PR remains.

When magnetic probes are added, plasma shape can be reconstructed with eddy current and sigma BT adjustment.





Detaches



Ipf4 reaches zero at t=1.42s.

21 FL

Inner boundary detaches from inboard limiter. Can be reconstructed even without eddy current.

21FL+5MP+1PR

Effect from inner PR becomes weakened due to the large number of MP?

When magnetic probes are added, plasma shape can be reconstructed with eddy current and sigma BT adjustment.

５. まとめと今後の予定

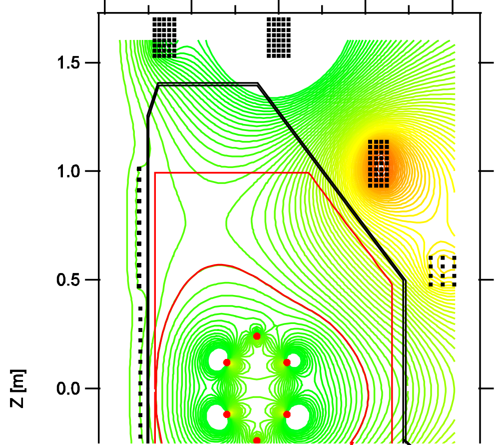
- CCS法
- 渦電流
- SVD, GCV
- ２種類の磁気計測

- 磁気プローブ増設（武智先生）
- 絶縁アンプ増設
- RT-ADC増設
- Reflective Memory
- 実時間表示（Alam留学生）

- 高速電子軌道計算（Alam留学生）
- ホロー電流分布平衡計算（清華大学）
- 非等方圧力分布平衡計算（福山先生）
- センサーレス反磁性測定結果の反映（飯尾先生）
- 誘導電流駆動（筒井先生）
- TV画像処理結果の反映（竹田先生）

This work was performed with the support and under the auspices of the NIFS Collaboration Research program (NIFS13KUTR096).

Shape reconstruction with two kinds of magnetic sensors

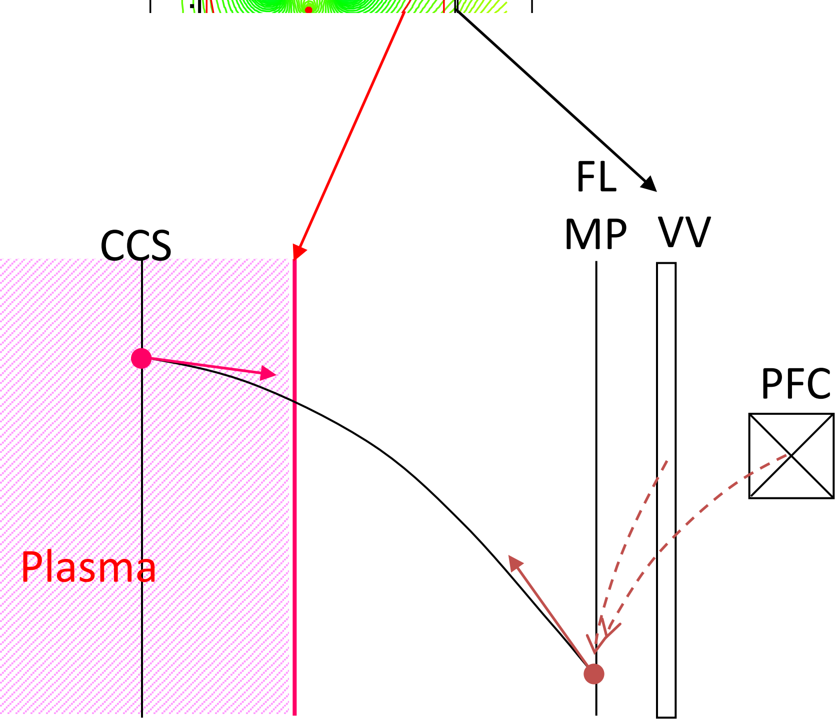


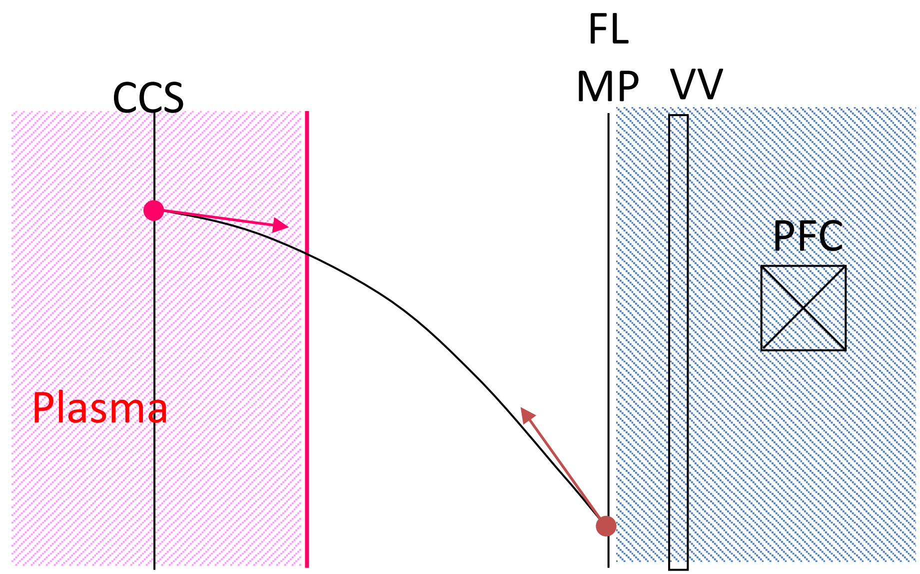
We may set CCS also on the magnetic sensor surface.

FL: Boundary condition of flux value

MP: Boundary condition of flux slope

We have only to consider eddy current and PF current only inside of CCS.

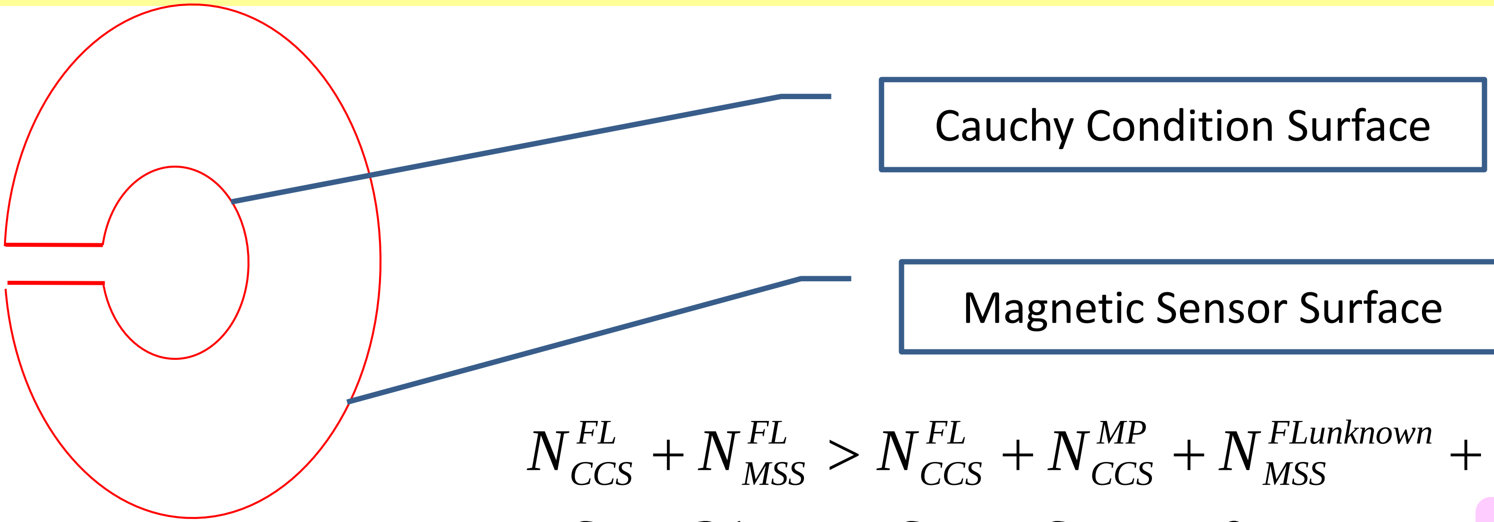




Only plasma is outside of vacuum field region. The CCS values are determined according to magnetic sensor signals. Boundary integral equation is applied only on the CCS.

Vacuum vessel and the outer space are also outside of vacuum region. Boundary integral equation is applied also on the magnetic sensor surface. Eddy current and PFC do not have to be considered.

Discretized equations of integral equations



$$N_{CCS}^{FL} + N_{MSS}^{FL} > N_{CCS}^{FL} + N_{CCS}^{MP} + N_{MSS}^{FLunknown} + N_{MSS}^{MPunknown}$$

$$6 \quad +21 \quad > \quad 6 \quad +6 \quad +0 \quad +(14-6)$$

Missing magnetic probes become unknown in stead of eddy currents.

(3) Flux value at point on CCS

$$\boxed{\frac{1}{2}\phi(\vec{x}_C)} = \sum_{i=1}^{M} W_{F3}(\vec{x}_C, \vec{z}_i)\boxed{\phi(\vec{z}_i)} + \sum_{i=1}^{M} W_{B3}(\vec{x}_C, \vec{z}_i)\boxed{Bt(\vec{z}_i)} + \cancel{\sum_{i=1}^{NE} W_{E3}(\vec{x}, \vec{y}_i)\boxed{E(\vec{y}_i)}} + \cancel{\sum_{j=1}^{NC} W_{C3}(\vec{x}_C, \vec{y}_j)\boxed{I_{PF}(\vec{y}_j)}}$$

Flux on CCS (unknown)

Flux on MSS (partly known, partly unknown)

Flux on CCS (unknown) and flux on MSS (partly known, partly unknown)

Magnetic field on CCS (unknown) and Magnetic field on MSS (partly known, partly unknown)

These terms don't have to be considered, since they are located outside of CCS.

6. 研究成果

[1] NAKAMURA Kazuo, et al., Shape Reconstruction of RF-Driven Divertor Plasma on QUEST, IEEE Transactions on Plasma Science, Vol. 42, No. 9, 2309-2312, 2014.09.
[2] KURIHARA Kenichi, ITAGAKI Masafumi, MIYATA Yoshiaki, NAKAMURA Kazuo, Special topic Article: The Cauchy Condition Surface (CCS) Method for Plasma Equilibrium Shape Reproduction, J. Plasma Fusion Res., Vol. 91, No. 1, 10-47, 2015.01.

[3] Kazuo Nakamura, et al., Reconstruction method of MHD equilibrium from two kinds of magnetic sensors, US-Japan Workshop on MHD, NIFS, Toki, 2014.03.11.
[4] Kazuo Nakamura, et al., Divertor plasma shape reconstruction from two kinds of magnetic sensors and eddy current effect on QUEST, SOFT 2014, 2014.10.02.
[5] Kazuo Nakamura, et al., Shape Reproduction and Particle Orbit in RF-Driven Divertor Plasma on QUEST, ITC-24,2014.11.04.
[6] Md. Mahbub Alam, et al., Particle Orbit in RF-Driven Divertor Plasma on QUEST, A3 Foresight Workshop on Spherical Torus, 2014.12.16.
[7] Kazuo Nakamura, et al., Shape Reproduction in RF-Driven Divertor Plasma on QUEST, A3 Foresight Workshop on Spherical Torus, 2014.12.16.

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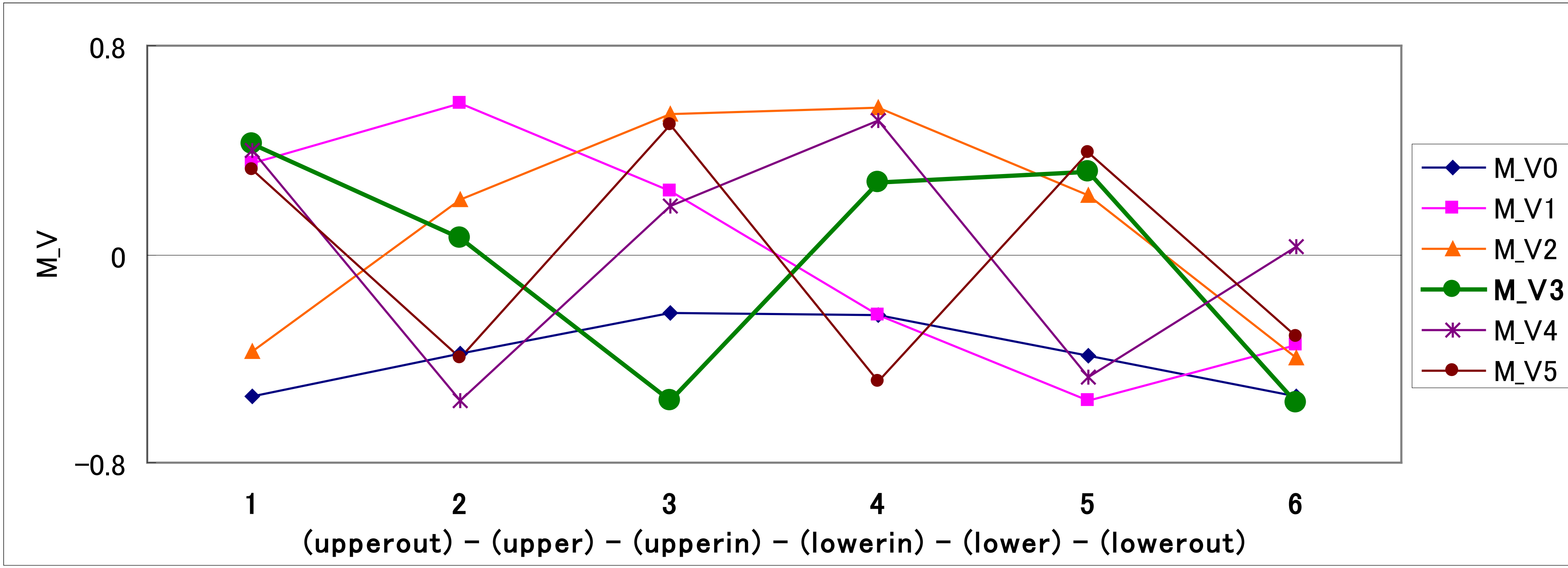
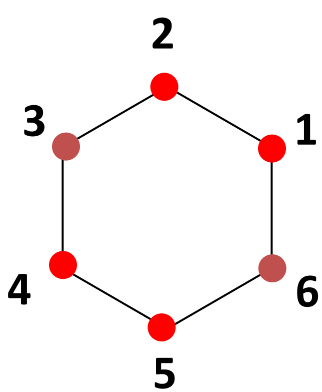
Singular Vectors of flux values on CCS





The fourth singular vector is NOT ODD with respect to the equatorial plane.